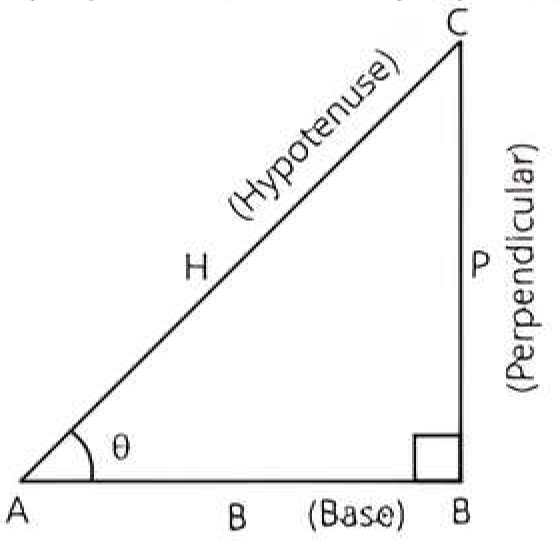
8 Introduction to Trigonometry

Fastrack Revision

- **Trigonometry:** It is the branch of Mathematics which deals with the measurement of angles and sides. 'Tri' means three, 'Gon' means sides and 'Metro' means measure.
- **Trigonometric Ratios:** Relations between different sides and angles of a right-angled triangle are called trigonometric ratios or T-ratios.
- In a right-angled ΔABC, $\angle B = 90°$, $\angle A = \theta$ (or $\angle C = 90° - \theta$) both are acute angles.
- Side opposite to right-angled $\angle B$ (90°) is known as hypotenuse (AC), side opposite to $\angle A$ (θ) is known as perpendicular (BC) and third side (AB) is known as base.



(i) $\sin\theta$ (sine of θ) $= \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{P}{H} = \frac{BC}{AC}$

(ii) $\cos\theta$ (cosine of θ) $= \frac{\text{Base}}{\text{Hypotenuse}} = \frac{B}{H} = \frac{AB}{AC}$

(iii) $\tan\theta$ (tangent of θ) $= \frac{\text{Perpendicular}}{\text{Base}} = \frac{P}{B} = \frac{BC}{AB}$

(iv) $\text{cosec}\,\theta$ (cosecant of θ) $= \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{H}{P} = \frac{AC}{AB}$

(v) $\sec\theta$ (secant of θ) $= \frac{\text{Hypotenuse}}{\text{Base}} = \frac{H}{B} = \frac{AC}{AB}$

(vi) $\cot\theta$ (cotangent of θ) $= \frac{\text{Base}}{\text{Perpendicular}} = \frac{B}{P} = \frac{AB}{BC}$

- **Relations between Trigonometric Ratios:**

(i) $\sin\theta = \frac{1}{\text{cosec}\,\theta}$ or $\text{cosec}\,\theta = \frac{1}{\sin\theta}$ or $\sin\theta \cdot \text{cosec}\,\theta = 1$

(ii) $\cos\theta = \frac{1}{\sec\theta}$ or $\sec\theta = \frac{1}{\cos\theta}$ or $\cos\theta \cdot \sec\theta = 1$

(iii) $\tan\theta = \frac{1}{\cot\theta}$ or $\cot\theta = \frac{1}{\tan\theta}$ or $\tan\theta \cdot \cot\theta = 1$

(iv) $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$

- If one of the trigonometric ratios of an acute angle is known, the remaining trigonometric ratios of the angle can be determined easily.
- **Pythagoras Theorem:** In a right-angled triangle, when two sides are given, then we find the third side using Pythagoras theorem.

$$\text{Hypotenuse}^2 = \text{Perpendicular}^2 + \text{Base}^2$$
$$H^2 = P^2 + B^2$$
or $$P^2 = H^2 - B^2 \quad \text{or} \quad B^2 = H^2 - P^2$$

- **Values of Trigonometric Ratios of Standard Angles**

T-Ratio \ θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
cot θ	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
cosec θ	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Knowledge BOOSTER

1. *The value of sin θ or cos θ never exceeds 1, whereas the value of sec θ or cosec θ is always greater than or equal to 1.*
2. *The values of the trigonometric ratios of an angle depend only on the magnitude of the angle and not on the lengths of the sides of the triangle.*

- **Trigonometric Identities:** Trigonometric identities are equalities that involve T-ratios and are true for every value of the occurring variables where both sides of the equality are defined.

(i) $\sin^2\theta + \cos^2\theta = 1$ for $0° \le \theta \le 90°$

or $\cos^2\theta = 1 - \cos^2\theta$
or $\cos^2\theta = 1 - \sin^2\theta$
} Conversions

(ii) $1+\tan^2\theta=\sec^2\theta$ for $0°\le\theta<90°$

$$\left.\begin{array}{ll}\text{or} & \tan^2\theta=\sec^2\theta-1\\ \text{or} & \sec^2\theta-\tan^2\theta=1\\ \text{or} & \tan^2\theta-\sec^2=-1\end{array}\right\}\text{Conversions}$$

(iii) $1+\cot^2\theta=\text{cosec}^2\theta$ for $0°<\theta\le90°$

$$\left.\begin{array}{ll}\text{or} & \cot^2\theta=\text{cosec}^2\theta-1\\ \text{or} & \text{cosec}^2\theta-\cot^2\theta=1\\ \text{or} & \cot^2\theta-\text{cosec}^2\theta=-1\end{array}\right\}\text{Conversions}$$

Practice Exercise

Multiple Choice Questions

Q 1. If $\cos A=\frac{4}{5}$ then the value of tan A is: [CBSE SQP 2023-24]

a. $\frac{3}{5}$ b. $\frac{3}{4}$

c. $\frac{4}{3}$ d. $\frac{1}{8}$

Q 2. Given that $\sin\theta=\frac{a}{b}$, find $\cos\theta$. [CBSE SQP 2023-24]

a. $\frac{b}{\sqrt{b^2-a^2}}$ b. $\frac{b}{a}$

c. $\frac{\sqrt{b^2-a^2}}{b}$ d. $\frac{a}{\sqrt{b^2-a^2}}$

Q 3. If $\cot\theta=\frac{1}{\sqrt{3}}$, then the value of $\sec^2\theta+\text{cosec}^2\theta$ is: [CBSE 2021 Term-I]

a. 1 b. $\frac{40}{9}$

c. $\frac{38}{9}$ d. $5\frac{1}{3}$

Q 4. Given that, $\sec\theta=\sqrt{2}$, the value of $\frac{1+\tan\theta}{\sin\theta}$ is: [CBSE 2021 Term-I]

a. $2\sqrt{2}$ b. $\sqrt{2}$

c. $3\sqrt{2}$ d. 2

Q 5. If $\tan A=\frac{2}{5}$, then the value of $\frac{1-\cos^2 A}{1-\sin^2 A}$ is: [CBSE 2023]

a. $\frac{25}{4}$ b. $\frac{4}{25}$

c. $\frac{4}{5}$ d. $\frac{5}{4}$

Q 6. If $5\tan\beta=4$, then $\frac{5\sin\beta-2\cos\beta}{5\sin\beta+2\cos\beta}=$ [CBSE SQP 2022-23]

a. $\frac{1}{3}$ b. $\frac{2}{5}$ c. $\frac{3}{5}$ d. 6

Q 7. If $2\cos\theta=1$, then the value of θ is: [CBSE 2023]

a. 45° b. 60°

c. 30° d. 90°

Q 8. If $\sqrt{3}\tan\theta=1$, then the value of θ is: [CBSE 2023]

a. 30° b. 45°

c. 60° d. 90°

Q 9. The value of $2\sin^2 30°+3\tan^2 60°-\cos^2 45°$ is: [CBSE 2023]

a. $3\sqrt{3}$ b. $\frac{19}{2}$

c. $\frac{9}{4}$ d. 9

Q 10. The value of $\frac{\sin 90°+\cos 60°}{\sec 45°+\tan 45°}$ is: [CBSE 2023]

a. 1 b. $\frac{3}{2}(\sqrt{2}+1)$

c. $\frac{3}{2}(\sqrt{2}-1)$ d. $\frac{1+\sqrt{3}}{\sqrt{2}+1}$

Q 11. If $x\tan 60°\cos 60°=\sin 60°\cot 60°$, then $x=$ [CBSE SQP 2022-23]

a. cos 30° b. tan 30°

c. sin 30° d. cot 30°

Q 12. If $\tan\alpha=\sqrt{3}$ and $\tan\beta=\frac{1}{\sqrt{3}}$, $0<\alpha,\beta<90°$, then the value of $\cot(\alpha+\beta)$ is:

a. $\sqrt{3}$ b. 0

c. $\frac{1}{\sqrt{3}}$ d. 1

Q 13. $\sin 2A=2\sin A$ is true when A is: [NCERT EXERCISE]

a. 0° b. 30°

c. 45° d. 60°

Q 14. $1-\cos^2 A$ is equal to: [CBSE SQP 2023-24]

a. $\sin^2 A$ b. $\tan^2 A$

c. $1-\sin^2 A$ d. $\sec^2 A$

Q 15. $8(\cos^2 A+\sin^2 A)$ is equal to: [CBSE 2023]

a. 1 b. 0

c. 9 d. 8

Q 16. $9\sec^2 A-9\tan^2 A$ is equal to: [CBSE 2023]

a. 9 b. 0

c. 8 d. $\frac{1}{9}$

Q 17. If θ is an acute angle of a right angled triangle, then which of the following equation is not true? *[CBSE 2023]*

a. $\sin\theta\cot\theta = \cos\theta$ b. $\cos\theta\tan\theta = \sin\theta$
c. $\operatorname{cosec}^2\theta - \cot^2\theta = 1$ d. $\tan^2\theta - \sec^2\theta = 1$

Q 18. $\dfrac{\cos^2\theta}{\sin^2\theta} - \dfrac{1}{\sin^2\theta}$ in simplified form is: *[CBSE 2023]*

a. $\tan^2\theta$ b. $\sec^2\theta$ c. 1 d. -1

Q 19. $(\sec^2\theta - 1)(\operatorname{cosec}^2\theta - 1)$ is equal to: *[CBSE 2023]*

a. -1 b. 1 c. 0 d. 2

Q 20. $(\sec A + \tan A)(1 - \sin A) =$ *[CBSE SQP 2023-24]*

a. sec A b. sin A c. cosec A d. cos A

Q 21. If $\sin\theta + \cos\theta = \sqrt{2}$, then $\tan\theta + \cot\theta =$ *[CBSE 2020, CBSE SQP 2022-23]*

a. 1 b. 2 c. 3 d. 4

Q 22. If $a\cot\theta + b\operatorname{cosec}\theta = p$ and $b\cot\theta + a\operatorname{cosec}\theta = q$, then $p^2 - q^2 =$ *[CBSE 2021 Term-I]*

a. $a^2 - b^2$ b. $b^2 - a^2$
c. $a^2 + b^2$ d. $b - a$

Q 23. Evaluate $\sqrt{\dfrac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\dfrac{\sec\theta + 1}{\sec\theta - 1}}$.

a. $2\sin\theta$ b. $2\cos\theta$
c. $2\operatorname{cosec}\theta$ d. $2\sec\theta$

Q 24. If $\sin A + \sin^2 A = 1$, then the value of the expression $(\cos^2 A + \cos^4 A)$ is: *[NCERT EXEMPLAR; CBSE 2020]*

a. 1 b. $\dfrac{1}{2}$ c. 2 d. 3

Q 25. If $2\sin^2\beta - \cos^2\beta = 2$, then β is: *[CBSE SQP 2021 Term-I]*

a. 0° b. 90° c. 45° d. 30°

Q 26. Which of the following is true for all values of θ ($0° \le \theta \le 90°$)? *[CBSE 2023]*

a. $\cos^2\theta - \sin^2\theta = 1$ b. $\operatorname{cosec}^2\theta - \sec^2\theta = 1$
c. $\sec^2\theta - \tan^2\theta = 1$ d. $\cot^2\theta - \tan^2\theta = 1$

Q 27. If $\operatorname{cosec}\theta - \cot\theta = \dfrac{1}{3}$, the value of $(\operatorname{cosec}\theta + \cot\theta)$ is: *[CBSE 2021 Term-I]*

a. 1 b. 2 c. 3 d. 4

Q 28. If $\sec\theta + \tan\theta = p$, then $\tan\theta$ is: *[CBSE 2021 Term-I]*

a. $\dfrac{p^2+1}{2p}$ b. $\dfrac{p^2-1}{2p}$

c. $\dfrac{p^2-1}{p^2+1}$ d. $\dfrac{p^2+1}{p^2-1}$

Assertion & Reason Type Questions

Directions (Q. Nos. 29-33): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
c. Assertion (A) is true but Reason (R) is false.
d. Assertion (A) is false but Reason (R) is true.

Q 29. Assertion (A): The value of each of the trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle, if the angle remains the same.
Reason (R): In right angled $\triangle ABC$, $\angle B = 90°$ and $\angle A = \theta$, $\sin\theta = \dfrac{BC}{AC} < 1$ and $\cos\theta = \dfrac{AB}{AC} < 1$ as hypotenuse is the longest side.

Q 30. Assertion (A): ABCD is a rectangle such that $\angle CAB = 60°$ and $AC = a$ units. The area of rectangle ABCD is $\dfrac{\sqrt{3}}{2}a^2$.

Reason (R): The value of $\sin 60°$ is $\dfrac{\sqrt{3}}{2}$ and $\cos 60°$ is $\dfrac{1}{2}$.

Q 31. Assertion (A): If $\sin\theta = \dfrac{1}{2}$ and θ is acute angle, then $(3\cos\theta - 4\cos^3\theta)$ is equal to 0.
Reason (R): As $\sin\theta = \dfrac{1}{2}$ and θ is acute, so θ must be 60°.

Q 32. Assertion (A): In a right-angled triangle, if $\tan\theta = \dfrac{3}{4}$, the greatest side of the triangle is 5 units.
Reason (R): $(\text{Greatest side})^2 = (\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$.

Q 33. Assertion (A): For $0° < \theta \le 90°$, $\operatorname{cosec}\theta - \cot\theta$ and $\operatorname{cosec}\theta + \cot\theta$ are reciprocal of each other.
Reason (R): $\operatorname{cosec}^2\theta - \cot^2\theta = 1$. *(CBSE 2023)*

Fill in the Blanks Type Questions

Q 34. The value of $\sin\theta$ or $\cos\theta$ never exceeds ....

Q 35. The minimum value of $\sec\theta$ is .... *[NCERT EXERCISE]*

Q 36. If $\sin\theta - \cos\theta = \dfrac{1}{4}$, then $\sin\theta \cdot \cos\theta =$ ....

True/False Type Questions

Q 37. The values of the trigonometric ratios of an angle depend only on the magnitude of the angle and not on the length of the sides of the triangle.

Q 38. The value of $\tan\theta$ increases from 0 to ∞ when θ increases from 0° to 90°.

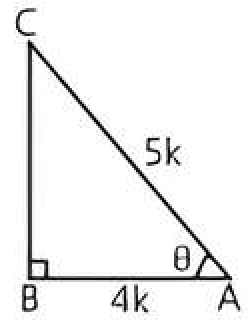
Q 39. One of the trigonometric identity is $\sec^2\theta - \tan^2\phi = 1$.

Q 40. If $\sin\theta = \dfrac{1}{2}$, then the value of $2\cot^2\theta + 2$ is 8.

Solutions

1. (b) Given, $\cos A = \frac{4}{5} = \frac{A}{K}$

Construct a right triangle ABC in which $\angle B = 90^\circ$ and base AB = 4k, hypotenuse $AC = 5k$, where k is a positive integer.



In right angled ΔABC,

$$AC^2 = AB^2 + BC^2$$

(By Pythagoras theorem)

$$\Rightarrow \quad (5k)^2 = (4k)^2 + BC^2$$

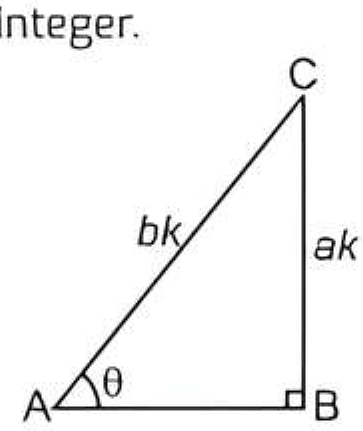
$$\Rightarrow \quad BC^2 = 25k^2 - 16k^2 = 9k^2$$

$$\therefore \quad BC = 3k$$

$$\therefore \quad \tan A = \frac{L}{A} = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}.$$

2. (c) Given, $\sin\theta = \frac{a}{b} = \frac{L}{K}$

Construct a right triangle ABC in which $\angle B = 90^\circ$ and perpendicular $BC = ak$, hypotenuse $AC = bk$, where k is a positive integer.



In right angled ΔABC,

$$AC^2 = AB^2 + BC^2 \text{ (by Pythagoras theorem)}$$

$$\Rightarrow \quad AB^2 = AC^2 - BC^2$$

$$= (bk)^2 - (ak)^2$$

$$= (b^2 - a^2)\,k^2$$

$$AB = k\sqrt{b^2 - a^2}$$

$$\therefore \quad \cos\theta = \frac{A}{K} = \frac{AB}{AC} = \frac{k\sqrt{b^2-a^2}}{bk} = \frac{\sqrt{b^2-a^2}}{b}$$

3. (d) Given, $\cot\theta = \frac{1}{\sqrt{3}}$

$$\Rightarrow \quad \cot\theta = \cot 60^\circ$$

$$\Rightarrow \quad \theta = 60^\circ$$

$$\therefore \quad \sec^2\theta + \csc^2\theta = \sec^2 60^\circ + \csc^2 60^\circ$$

$$= (2)^2 + \left(\frac{2}{\sqrt{3}}\right)^2 = 4 + \frac{4}{3}$$

$$= \frac{16}{3} = 5\frac{1}{3}$$

4. (a) Given, $\sec\theta = \sqrt{2}$

$$\Rightarrow \quad \sec\theta = \sec 45^\circ$$

$$\Rightarrow \quad \theta = 45^\circ$$

$$\therefore \quad \frac{1+\tan\theta}{\sin\theta} = \frac{1+\tan 45^\circ}{\sin 45^\circ}$$

$$= \frac{1+1}{\frac{1}{\sqrt{2}}} = \frac{2\times\sqrt{2}}{1} = 2\sqrt{2}$$

5. (b) Given, $\tan A = \frac{2}{5}$

> **TiP**
>
> $\sin^2 A + \cos^2 A = 1$
>
> *or* $\quad 1 - \sin^2 A = \cos^2 A$
>
> *or* $\quad 1 - \cos^2 A = \sin^2 A$

$$\therefore \quad \frac{1-\cos^2 A}{1-\sin^2 A} = \frac{\sin^2 A}{\cos^2 A}$$

$$= \left(\frac{\sin A}{\cos A}\right)^2 = (\tan A)^2 = \left(\frac{2}{5}\right)^2 = \frac{4}{25}.$$

6. (a) Given, $5\tan\beta = 4 \Rightarrow \tan\beta = \frac{4}{5}$

$$\therefore \quad \frac{5\sin\beta - 2\cos\beta}{5\sin\beta + 2\cos\beta} = \frac{5\tan\beta - 2}{5\tan\beta + 2}$$

$$= \frac{5\times\frac{4}{5} - 2}{5\times\frac{4}{5} + 2} = \frac{4-2}{4+2} = \frac{2}{6} = \frac{1}{3}$$

7. (b) Given, $2\cos\theta = 1$

$$\Rightarrow \quad \cos\theta = \frac{1}{2}$$

$$\Rightarrow \quad \cos\theta = \cos 60^\circ$$

$$\therefore \quad \theta = 60^\circ$$

8. (a) Given, $\sqrt{3}\tan\theta = 1$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \tan\theta = \tan 30^\circ$$

$$\therefore \quad \theta = 30^\circ$$

9. (d) $2\sin^2 30^\circ + 3\tan^2 60^\circ - \cos^2 45^\circ$

$$= 2\left(\frac{1}{2}\right)^2 + 3(\sqrt{3})^2 - \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 2\times\frac{1}{4} + 3\times 3 - \frac{1}{2}$$

$$= \frac{1}{2} + 9 - \frac{1}{2} = 9.$$

10. (c) $\frac{\sin 90^\circ + \cos 60^\circ}{\sec 45^\circ + \tan 45^\circ}$

$$= \frac{1+\frac{1}{2}}{\sqrt{2}+1} = \frac{3}{2(\sqrt{2}+1)}$$

$$= \frac{3}{2(\sqrt{2}+1)} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)}$$

$$= \frac{3(\sqrt{2}-1)}{2\{(\sqrt{2})^2-(1)^2\}} = \frac{3(\sqrt{2}-1)}{2(2-1)}$$

$$= \frac{3}{2}(\sqrt{2}-1).$$

11. (b) Given,

$x \tan 60^\circ \cos 60^\circ = \sin 60^\circ \cot 60^\circ$

$\therefore \quad x \times \sqrt{3} \times \frac{1}{2} = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}}$

$\Rightarrow \quad x = \frac{1}{\sqrt{3}}$

$= \tan 30^\circ$

12. (b) We have, $\tan\alpha = \sqrt{3}$

$\Rightarrow \quad \alpha = 60^\circ > 0^\circ \quad (\because \tan 60^\circ = \sqrt{3})$

and $\quad \tan\beta = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \beta = 30^\circ < 90^\circ \quad \left(\because \tan 30^\circ = \frac{1}{\sqrt{3}}\right)$

$\therefore \quad \alpha + \beta = 60^\circ + 30^\circ = 90^\circ$

$\therefore \quad \cot(\alpha+\beta) = \cot 90^\circ = 0$

13. (a) From option (a).

$LHS = \sin 2A = \sin 2 \times 0^\circ = \sin 0^\circ = 0$

and $\quad RHS = 2 \sin A = 2 \sin 0^\circ$

$= 2 \times 0 = 0$

$\because \quad LHS = RHS$

$\therefore \quad A = 0^\circ$

14. (a) We know that $\sin^2 A + \cos^2 A = 1$

$\therefore \quad \sin^2 A = 1 - \cos^2 A$

or $\quad 1 - \cos^2 A = \sin^2 A.$

15. (d) $8(\cos^2 A + \sin^2 A)$

$= 8(\sin^2 A + \cos^2 A)$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= 8 \times 1 = 8$

16. (a) $9\sec^2 A - 9\tan^2 A$

$= 9(\sec^2 A - \tan^2 A)$

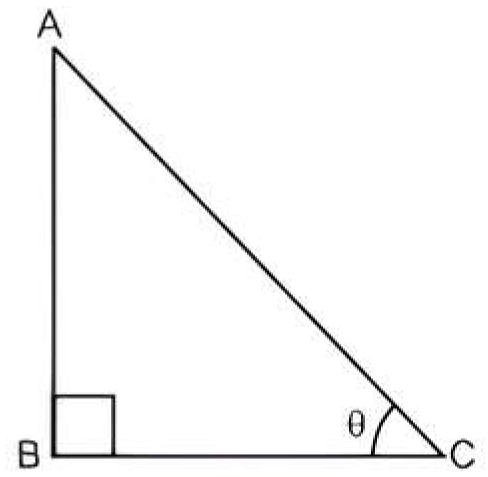
$[\because \sec^2\theta - \tan^2\theta = 1]$

$= 9 \times 1 = 9$

17. (d) Given, $\quad \theta$ = acute angle

Let $\quad \angle B = 90^\circ$

$\therefore$ In right angled ΔABC.



$AC^2 = AB^2 + BC^2$ (by Pythagoras theorem) ...(1)

Option (a): L.H.S. $= \sin\theta \cdot \cot\theta$

$$= \frac{AB}{AC} \times \frac{BC}{AB} = \frac{BC}{AC} = \cos\theta = \text{R.H.S.}$$

Option (b): L.H.S. $= \cos\theta \cdot \tan\theta$

$$= \frac{BC}{AC} \times \frac{AB}{BC} = \frac{AB}{AC} = \sin\theta = \text{R.H.S.}$$

Option (c): L.H.S. $= \text{cosec}^2\theta - \cot^2\theta$

$$= \left(\frac{AC}{AB}\right)^2 - \left(\frac{BC}{AB}\right)^2 = \frac{AC^2}{AB^2} - \frac{BC^2}{AB^2}$$

$$= \frac{AC^2 - BC^2}{AB^2} = \frac{AB^2}{AB^2} = 1 \quad \text{[from eq. (1)]}$$

$= \text{R.H.S.}$

Option (d): L.H.S. $= \tan^2\theta - \sec^2\theta$

$$= \left(\frac{AB}{BC}\right)^2 - \left(\frac{AC}{BC}\right)^2 = \frac{AB^2}{BC^2} - \frac{AC^2}{BC^2}$$

$$= \frac{AB^2 - AC^2}{BC^2} = \frac{-BC^2}{BC^2} = -1 \quad \text{[from eq. (1)]}$$

$\neq \text{R.H.S.}$

18. (d) $\frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta} = \frac{\cos^2\theta - 1}{\sin^2\theta} \quad (\because \sin^2\theta + \cos^2\theta = 1)$

$$= \frac{-\sin^2\theta}{\sin^2\theta} = -1$$

19. (b) $(\sec^2\theta - 1)(\text{cosec}^2\theta - 1) \left(\begin{array}{l}\because 1+\tan^2\theta = \sec^2\theta \text{ and} \\ 1+\cot^2\theta = \text{cosec}^2\theta\end{array}\right)$

$= \tan^2\theta \cdot \cot^2\theta$

$$= \frac{\sin^2\theta}{\cos^2\theta} \times \frac{\cos^2\theta}{\sin^2\theta} = 1.$$

20. (d) $(\sec A + \tan A)(1 - \sin A)$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{(1+\sin A)}{\cos A} \times (1 - \sin A) = \frac{1 - \sin^2 A}{\cos A}$$

$(\because \sin^2 A + \cos^2 A = 1)$

$$= \frac{\cos^2 A}{\cos A} = \cos A$$

21. (b) Given, $\sin\theta + \cos\theta = \sqrt{2}$

Squaring both sides,

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$

$\Rightarrow \quad 1 + 2\sin\theta\cos\theta = 2$

$\Rightarrow \quad 2\sin\theta\cos\theta = 1$

$\Rightarrow \quad \sin\theta\cos\theta = \frac{1}{2}$ ...(1)

Now, $\quad \tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{1/2} \quad \text{[from eq. (1)]}$$

$= 2$

22. (b) We have, $a \cot\theta + b \operatorname{cosec}\theta = p$ ...(1)
and $b \cot\theta + a \operatorname{cosec}\theta = q$ ...(2)
Squaring and then subtracting eq. (2) from eq. (1), we get

$$p^2 - q^2 = a^2 \cot^2\theta + b^2 \operatorname{cosec}^2\theta + 2ab \cot\theta \operatorname{cosec}\theta - b^2\cot^2\theta - a^2\operatorname{cosec}^2\theta - 2ab\cot\theta\operatorname{cosec}\theta$$
$$= a^2(\cot^2\theta - \operatorname{cosec}^2\theta) + b^2(\operatorname{cosec}^2\theta - \cot^2\theta)$$
$$= -a^2 + b^2 \quad (\because \operatorname{cosec}^2\theta - \cot^2\theta = 1)$$
$$= b^2 - a^2$$

23. (c) We have, $\sqrt{\dfrac{\sec\theta-1}{\sec\theta+1}} + \sqrt{\dfrac{\sec\theta+1}{\sec\theta-1}}$

$$= \frac{(\sec\theta-1)+(\sec\theta+1)}{\sqrt{\sec^2\theta-1}} = \frac{2\sec\theta}{\sqrt{\tan^2\theta}} \quad (\because \sec^2\theta - 1 = \tan^2\theta)$$

$$= \frac{2\sec\theta}{\tan\theta} = \frac{2}{\cos\theta}\times\frac{\cos\theta}{\sin\theta} = \frac{2}{\sin\theta} = 2\operatorname{cosec}\theta$$

24. (a) Given, $\sin A + \sin^2 A = 1$

> **TiP**
> *Adequate practice of identities is necessary to avoid errors in simplification.*

$$\Rightarrow \sin A = 1 - \sin^2 A = \cos^2 A \quad (\because \cos^2 A + \sin^2 A = 1)$$

Squaring both sides, we get

$$\sin^2 A = \cos^4 A$$
$$1 - \cos^2 A = \cos^4 A \Rightarrow \cos^2 A + \cos^4 A = 1$$

> **TiP**
> *Sometimes students forget the identity used and hence commit error.*

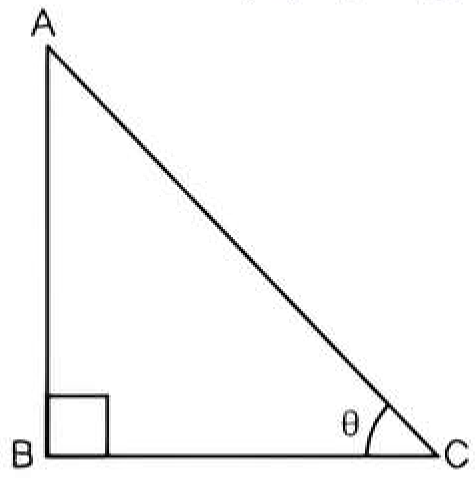
25. (b) Given, $2\sin^2\beta - \cos^2\beta = 2$

$$\Rightarrow -\cos^2\beta = 2(1-\sin^2\beta)$$
$$\Rightarrow -\cos^2\beta = 2\cos^2\beta \quad (\because \sin^2\theta + \cos^2\theta = 1)$$
$$\Rightarrow 3\cos^2\beta = 0$$
$$\Rightarrow \cos^2\beta = 0$$
$$\Rightarrow \cos\beta = 0 = \cos 90^\circ$$
$$\therefore \beta = 90^\circ$$

26. (c) Let $\angle B = 90^\circ$ and $0^\circ \le \theta \le 90^\circ$

$\therefore$ In right ΔABC,

$AC^2 = AB^2 + BC^2$ (by Pythagoras theorem) ...(1)



Option (a): L.H.S. $= \cos^2\theta - \sin^2\theta$

$$= \left(\frac{BC}{AC}\right)^2 - \left(\frac{AB}{AC}\right)^2 = \frac{BC^2 - AB^2}{AC^2} \neq \text{R.H.S.}$$

Option (b): L.H.S. $= \operatorname{cosec}^2\theta - \sin^2\theta$

$$= \left(\frac{AC}{AB}\right)^2 - \left(\frac{AC}{BC}\right)^2$$

$$= \frac{AC^2(BC^2 - AB^2)}{AB^2 \cdot BC^2} \neq \text{R.H.S.}$$

Option (c): L.H.S. $= \sec^2\theta - \tan^2\theta$

$$= \left(\frac{AC}{BC}\right)^2 - \left(\frac{AB}{BC}\right)^2$$

$$= \frac{AC^2 - AB^2}{BC^2}$$

$$= \frac{BC^2}{BC^2} \quad \text{[from eq. (1)]}$$

$$= 1 = \text{R.H.S.}$$

Option (d): L.H.S. $= \cot^2\theta - \tan^2\theta$

$$= \left(\frac{BC}{AB}\right)^2 - \left(\frac{AB}{BC}\right)^2 = \frac{BC^4 - AB^4}{AB^2 \cdot BC^2}$$

$$= \text{R.H.S.}$$

27. (c) Given, $\operatorname{cosec}\theta - \cot\theta = \dfrac{1}{3}$

Also, $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$$\Rightarrow (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 1$$
$$\Rightarrow (\operatorname{cosec}\theta + \cot\theta)\times\frac{1}{3} = 1$$
$$\Rightarrow \operatorname{cosec}\theta + \cot\theta = 3$$

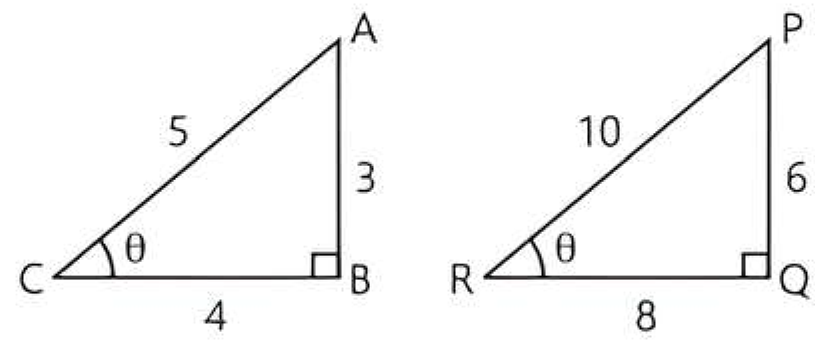
28. (b) Given, $\sec\theta + \tan\theta = p$ .... (1)

$$\because \sec^2\theta - \tan^2\theta = 1$$
$$\Rightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1 \quad [\because a^2 - b^2 = (a-b)(a+b)]$$
$$\Rightarrow (\sec\theta - \tan\theta)\,p = 1$$
$$\Rightarrow \sec\theta - \tan\theta = \frac{1}{p} \quad .... (2)$$

Subtracting eq. (2) from eq. (1), we get

$$2\tan\theta = p - \frac{1}{p}$$
$$\Rightarrow \tan\theta = \frac{p^2-1}{2p}$$

29. (b) **Assertion (A):** Suppose in ΔABC and in ΔPQR



$$\sin\theta = \frac{AB}{AC} = \frac{3}{5} \text{ and } \sin\theta = \frac{PQ}{PR} = \frac{6}{10}$$

$$\Rightarrow \sin\theta = \frac{3}{5} \text{ and } \sin\theta = \frac{3}{5}$$

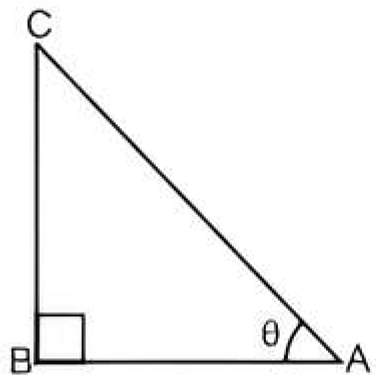
Similarly, this will also holds for other trigonometric ratios.

So, trigonometric ratio does not depend on the size of the triangle.

So, Assertion (A) is true.

Reason (R): Given, $\angle B = 90°$, $\angle A = \theta$ and

$\sin\theta = \dfrac{BC}{AC} < 1$



and $\cos\theta = \dfrac{AB}{AC} < 1$

$\therefore \quad \sin^2\theta + \cos^2\theta < 1$

$\Rightarrow \quad \left(\dfrac{BC}{AC}\right)^2 + \left(\dfrac{AB}{AC}\right)^2 < 1$

$\Rightarrow \quad \dfrac{BC^2}{AC^2} + \dfrac{AB^2}{AC^2} < 1$

$\Rightarrow \quad AB^2 + BC^2 < AC^2$

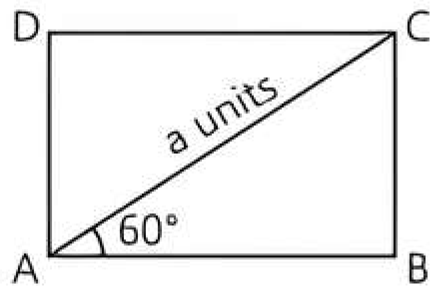
So, in right ΔABC hypotenuse is the longest side.

$\therefore$ Reason (R) is also true.

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

30. (d) **Assertion (A):** In ΔABC, $AC = a$ units, $\angle A = 60°$

$\therefore \quad \sin 60° = \dfrac{BC}{AC} = \dfrac{BC}{a}$



$\Rightarrow \quad \dfrac{\sqrt{3}}{2} = \dfrac{BC}{a} \quad \Rightarrow BC = \dfrac{a\sqrt{3}}{2}$

Also, $\cos 60° = \dfrac{AB}{AC} \quad \Rightarrow \dfrac{1}{2} = \dfrac{AB}{a}$

$\Rightarrow \quad AB = a/2$

$\therefore$ Area of rectangle ABCD = AB × BC

$$= \frac{a}{2} \times \frac{a\sqrt{3}}{2} = \frac{\sqrt{3}}{4}a^2$$

So, Assertion (A) is false.

Reason (R): It is true to say that the value of sin 60° is $\dfrac{\sqrt{3}}{2}$ and cos 60° is $\dfrac{1}{2}$.

Hence, Assertion (A) is false but Reason (R) is true.

31. (c) **Assertion (A):** We have, $\sin\theta = \dfrac{1}{2}$

$\Rightarrow \quad \theta = 30° \quad \left(\because \sin 30° = \dfrac{1}{2}\right)$

$\therefore \quad 3\cos\theta - 4\cos^3\theta = 3\cos 30° - 4\cos^3 30°$

$$= \frac{3\sqrt{3}}{2} - 4\left(\frac{\sqrt{3}}{2}\right)^3 = \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0$$

So, Assertion (A) is true.

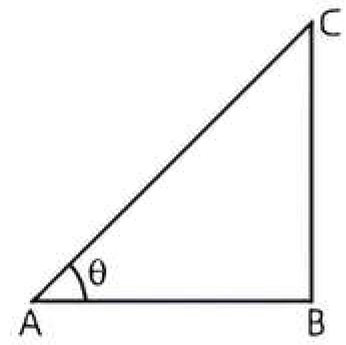
Reason (R): It is false to say that at $\theta = 60°$, $\sin\theta = \dfrac{1}{2}$.

This will be correct at $\theta = 30°$.

Hence, Assertion (A) is true but Reason (R) is false.

32. (a) **Assertion (A):** Given,

$$\tan\theta = \frac{3}{4} = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB}$$



Let $BC = 3k$ and $AB = 4k$

In right-angled ΔABC, by Pythagoras theorem,

$$AC = \sqrt{(AB)^2 + (BC)^2}$$
$$= \sqrt{(4k)^2 + (3k)^2} = \sqrt{16k^2 + 9k^2}$$
$$= \sqrt{25k^2} = 5k$$

It is true to say that greatest side of a triangle is hypotenuse.

So, Assertion (A) is true.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

33. (a) **Assertion (A):** For $0° < \theta \le 90°$, $\text{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow \quad (\text{cosec}\,\theta - \cot\theta)(\text{cosec}\,\theta + \cot\theta) = 1$

$[\because a^2 - b^2 = (a-b)(a+b)]$

$\Rightarrow \quad \text{cosec}\,\theta - \cot\theta = \dfrac{1}{\text{cosec}\,\theta + \cot\theta}$

Or

$\text{cosec}\,\theta + \cot\theta = \dfrac{1}{\text{cosec}\,\theta - \cot\theta}$

$\therefore (\text{cosec}\,\theta - \cot\theta)$ and $(\text{cosec}\,\theta + \cot\theta)$ are reciprocal of each other.

So, Assertion (A) is true.

Reason (R): It is also true.

Hence both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

34. one

35. The minimum value of sec θ is one.

36. Given, $\sin\theta - \cos\theta = \dfrac{1}{4}$

Squaring on both sides, we get

$$(\sin\theta - \cos\theta)^2 = \left(\frac{1}{4}\right)^2$$

$\Rightarrow \quad \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta = \dfrac{1}{16}$

$\Rightarrow \quad 1 - 2\sin\theta\cos\theta = \dfrac{1}{16}$

$\Rightarrow \quad 2\sin\theta\cos\theta = 1 - \dfrac{1}{16}$

$\Rightarrow \quad \sin\theta\cos\theta = \dfrac{1}{2}\left(\dfrac{15}{16}\right) = \dfrac{15}{32}$

37. True

38. True

39. In the given equation, both trigonometric ratios have different angles. So, the given equation will not have trigonometric identity.
Hence, given statement is false.

40. Given, $\sin\theta = \frac{1}{2}$

$$\Rightarrow \quad \theta = 30^\circ$$

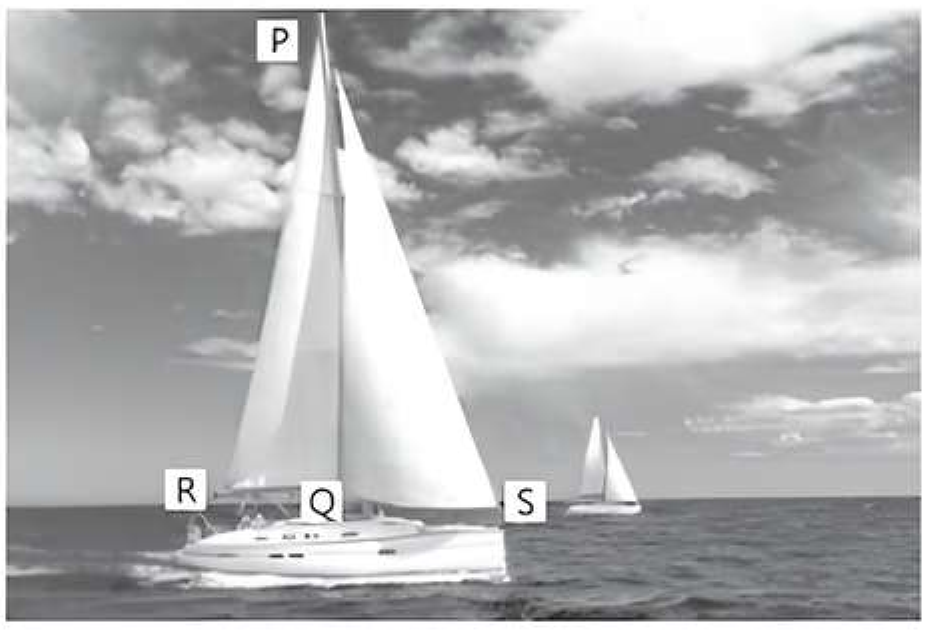
$$\therefore \quad 2\cot^2\theta + 2 = 2\cot^2 30^\circ + 2$$
$$= 2(\sqrt{3})^2 + 2$$
$$= 2 \times 3 + 2$$
$$= 6 + 2 = 8$$

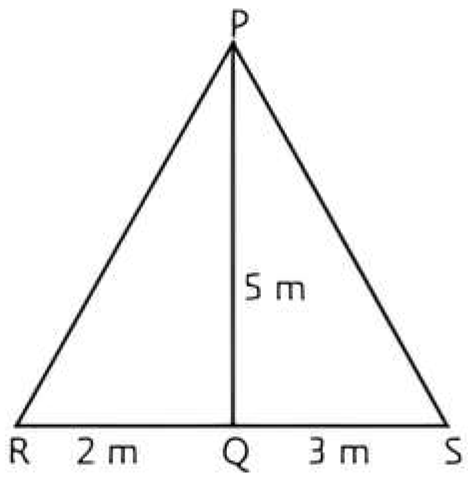
Hence, given statement is true.

Case Study Based Questions

Case Study 1

A sailing boat with triangular masts is as shown below. Two right triangles can be observed Triangles PQR and PQS, both right-angled at Q. The distance QR = 2 m and QS = 3 m and height PQ = 5 m.





Based on the above information, solve the following questions:

Q 1. The value of sec S is:

a. $\frac{\sqrt{34}}{5}$ b. $\frac{\sqrt{34}}{3}$

c. $\frac{5}{3}$ d. $\frac{3}{\sqrt{34}}$

Q 2. The value of cosec R is:

a. $\frac{\sqrt{29}}{5}$ b. $\frac{\sqrt{29}}{2}$

c. $\frac{2}{5}$ d. $\frac{5}{\sqrt{29}}$

Q 3. The value of tan S + cot R is:

a. $\frac{9}{4}$ b. $\frac{5}{3}$ c. $\frac{31}{15}$ d. $\frac{9}{5}$

Q 4. The value of $\sin^2 R - \cos^2 S$ is:

a. 0 b. 1 c. $\frac{97}{85}$ d. $\frac{589}{986}$

Q 5. The value of $\sin^2 S + \cos^2 R$ is:

a. 0 b. 1

c. $\frac{97}{85}$ d. $\frac{861}{986}$

Solutions

1. In right-angled ΔPQS

TR!CK

In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$$(PS)^2 = (SQ)^2 + (PQ)^2 = (3)^2 + (5)^2 = 9 + 25 = 34$$

(by Pythagoras theorem)

$$\Rightarrow \quad PS = \sqrt{34}\,m$$

$\therefore$ In right-angled ΔPQS,

$$\sec S = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{PS}{SQ} = \frac{\sqrt{34}}{3}$$

So, option (b) is correct.

2. In right-angled ΔPQR

$$(PR)^2 = (PQ)^2 + (QR)^2$$

(by Pythagoras theorem)

$$= (5)^2 + (2)^2 = 25 + 4$$
$$= 29$$

$$\Rightarrow \quad PR = \sqrt{29}\,m$$

$\therefore$ In right-angled ΔPQR,

$$\text{cosec } R = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{PR}{PQ} = \frac{\sqrt{29}}{5}$$

So, option (a) is correct.

3. Use the identity,

$$1 + \tan^2 S = \sec^2 S$$

$$\Rightarrow \quad \tan S = \sqrt{\sec^2 S - 1} = \sqrt{\left(\frac{\sqrt{34}}{3}\right)^2 - 1}$$

(from part 1)

$$= \sqrt{\frac{34}{9} - 1} = \sqrt{\frac{25}{9}} = \frac{5}{3}$$

Use the identity, $1 + \cot^2 R = \text{cosec}^2 R$

$$\Rightarrow \quad \cot R = \sqrt{\text{cosec}^2 R - 1} = \sqrt{\left(\frac{\sqrt{29}}{5}\right)^2 - 1}$$

(from part 2)

$$= \sqrt{\frac{29}{25} - 1} = \sqrt{\frac{4}{25}} = \frac{2}{5}$$

$\therefore \quad \tan S + \cot R = \frac{5}{3} + \frac{2}{5} = \frac{25+6}{15} = \frac{31}{15}$

So, option (c) is correct.

4. From part (1), $\sec S = \frac{\sqrt{34}}{3}$

$\Rightarrow \quad \cos S = \frac{3}{\sqrt{34}}$

TiP

$\cos\theta = \frac{1}{\sec\theta}, \sin\theta = \frac{1}{\text{cosec}\,\theta}$

From part (2), $\text{cosec}\,R = \frac{\sqrt{29}}{5}$

$\Rightarrow \quad \sin R = \frac{5}{\sqrt{29}}$

$\therefore \sin^2 R - \cos^2 S = \left(\frac{5}{\sqrt{29}}\right)^2 - \left(\frac{3}{\sqrt{34}}\right)^2 = \frac{25}{29} - \frac{9}{34}$

$= \frac{850-261}{986} = \frac{589}{986}$

So, option (d) is correct.

5. From part (1), $\sec S = \frac{\sqrt{34}}{3}$

$\Rightarrow \quad \cos S = \frac{3}{\sqrt{34}}$

$\therefore \quad \sin S = \sqrt{1-\cos^2 S} = \sqrt{1-\frac{9}{34}} = \sqrt{\frac{25}{34}} = \frac{5}{\sqrt{34}}$

From part (2), $\text{cosec}\,R = \frac{\sqrt{29}}{5}$

$\Rightarrow \quad \sin R = \frac{5}{\sqrt{29}}$

$\cos R = \sqrt{1-\sin^2 R} = \sqrt{1-\frac{25}{29}} = \sqrt{\frac{4}{29}} = \frac{2}{\sqrt{29}}$

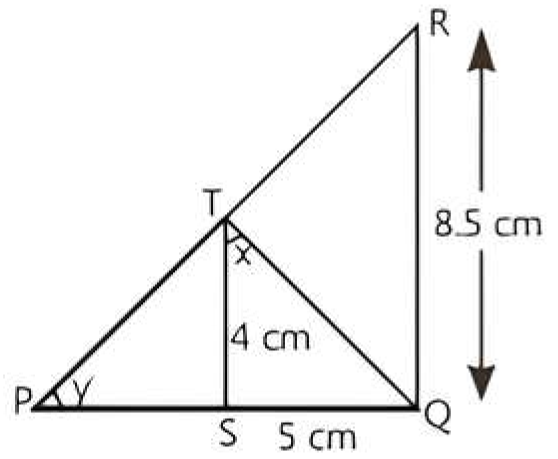
$\sin^2 S + \cos^2 R = \left(\frac{5}{\sqrt{34}}\right)^2 + \left(\frac{2}{\sqrt{29}}\right)^2 = \frac{25}{34} + \frac{4}{29}$

$= \frac{725+136}{986} = \frac{861}{986}$

So, option (d) is correct.

Case Study 2

Anika is studying in X standard. She is making a figure to understand trigonometric ratio shown as below.



In ΔPQR, $\angle Q$ is a right angle, ΔQTR is right-angled at T and ΔQST is right-angled at S, PQ = 12 cm, QR = 8.5 cm, ST = 4 cm, SQ = 5 cm, $\angle QTS = x$ and $\angle TPQ = y$.

Based on the given information, solve the following questions:

Q 1. The length of PT is:

a. 8 cm
b. $\sqrt{65}$ cm
c. 7.5 cm
d. $\sqrt{69}$ cm

Q 2. The value of tan x is:

a. $\frac{7.5}{13}$
b. $\frac{5}{4}$
c. $\frac{4}{5}$
d. $\frac{13}{7.5}$

Q 3. The value of sec x is:

a. $\frac{\sqrt{91}}{6}$
b. $\frac{\sqrt{71}}{6}$
c. $\frac{\sqrt{41}}{4}$
d. $\frac{\sqrt{31}}{5}$

Q 4. The value of sin y is:

a. $\frac{4}{\sqrt{65}}$
b. $\frac{4}{7}$
c. $\frac{7}{4}$
d. $\frac{\sqrt{65}}{7}$

Q 5. The value of cot y is:

a. $\frac{7}{4}$
b. $\frac{4}{7}$
c. $\frac{\sqrt{65}}{4}$
d. $\frac{\sqrt{65}}{7}$

Solutions

1. We have, PS = PQ − SQ = 12 − 5 = 7 cm

TR!CK

In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

In right-angled ΔPST,

$(PT)^2 = (PS)^2 + (ST)^2$ (By Pythagoras theorem)

$= (7)^2 + (4)^2 = 49 + 16 = 65$

$\Rightarrow PT = \sqrt{65}$ cm

So, option (b) is correct.

2. In right-angled ΔTSQ,

$\tan x = \frac{\text{Perpendicular}}{\text{Base}} = \frac{SQ}{TS} = \frac{5}{4}$

So, option (b) is correct.

3. We know the identity.

$$\sec^2 x = 1 + \tan^2 x = 1 + \left(\frac{5}{4}\right)^2 = 1 + \frac{25}{16} = \frac{41}{16}$$

$$\Rightarrow \quad \sec x = \sqrt{\frac{41}{16}} = \frac{\sqrt{41}}{4}$$

So, option (c) is correct.

4. In right-angled ΔTSP,

$$\sin y = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{TS}{PT} = \frac{4}{\sqrt{65}}$$

So, option (a) is correct.

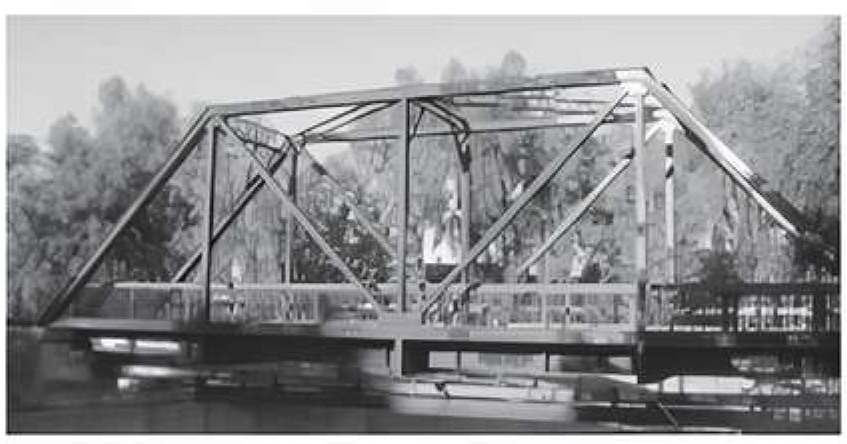
5. In right angled ΔTSP

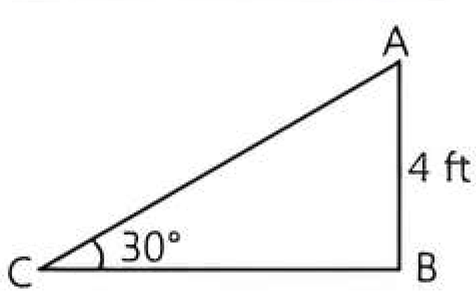
$$\cot y = \frac{\text{Base}}{\text{Perpendicular}} = \frac{PS}{TS} = \frac{7}{4}$$

So, option (a) is correct.

Case Study 3

In structural design, a structure is composed of triangles that are interconnecting. A truss a series of triangle in same plane end is one of the major types of engineering structures and is especially used in the design of bridges and buildings. Trusses are designed to support loads, such as the weight of people. A truss is exclusively made of long, straight members connected by joints at the end of each member.





This is a single repeating triangle in a truss system.

Based on the above information, solve the following questions:

Q 1. In the above triangle, what is the length of AC?

Q 2. In the above triangle, what is the length of BC?

Q 3. If sin A = sin C, what will be the length of BC?

Or

If the length of AB doubles, what will happen the length of AC?

Solutions

1. In right angled ΔABC,

$$\sin 30° = \frac{AB}{AC} \Rightarrow \frac{1}{2} = \frac{4}{AC}$$

$$\Rightarrow \quad AC = 8 \text{ ft}$$

2. In right-angled ΔABC,

$$\tan 30° = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{4}{BC} \Rightarrow BC = 4\sqrt{3} \text{ ft}$$

3. Given, $\sin A = \sin C$

In right-angled ΔABC,

$$\frac{BC}{AC} = \frac{AB}{AC} \Rightarrow BC = AB = 4 \text{ ft}$$

Or

Given, $AB = 2 \times 4 = 8$ ft

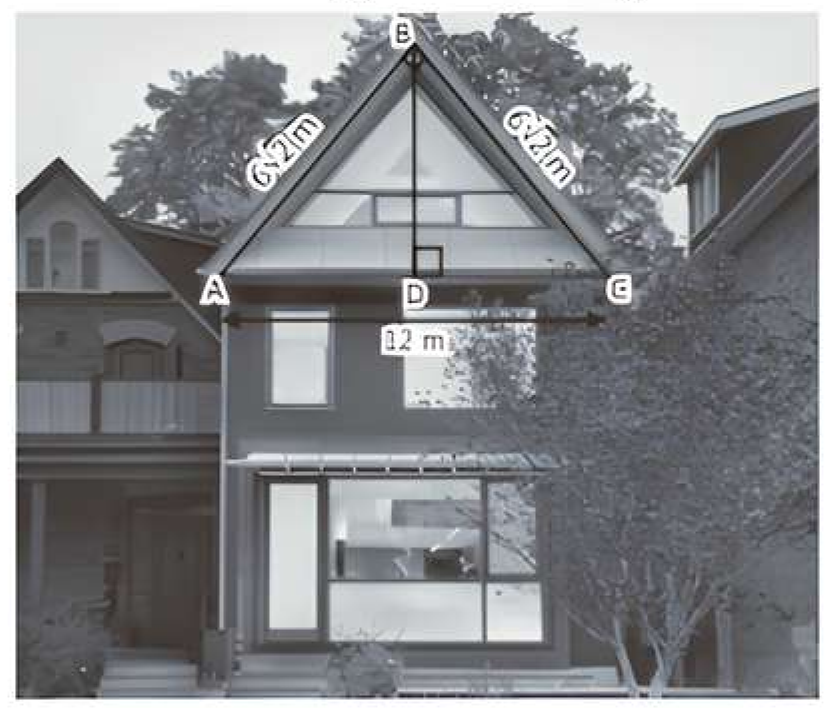
$$\therefore \text{In right } \Delta ABC, \sin 30° = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{8}{AC} \Rightarrow AC = 16 \text{ ft}$$

So, AC doubles the original length.

Case Study 4

Soniya and her father went to her friend Ruhi to enjoy party. When they reached Ruhi's place, Soniya saw the roof of the house, which was triangular in shape. She imagined the dimensions of the roof which is as given in the figure.



Based on the above information, solve the following questions:

Q 1. If D is the mid-point of AC, then find BD.

Q 2. Find the measure of ∠A and ∠C.

Q 3. Find the value of sin A + cos C.

Or

Find the value of $\tan^2 C + \tan^2 A$.

Solutions

1. We have, $AB = BC = 6\sqrt{2}$ m and $AC = 12$ m

$\because$ D is the mid-point of AC.

$$\therefore \quad AD = DC = \frac{12}{2} = 6 \text{ m}$$

In right-angled ΔADB, use Pythagoras theorem

$$AB^2 = BD^2 + AD^2$$

$$\Rightarrow \quad BD^2 = (6\sqrt{2})^2 - 6^2$$

$$BD^2 = 72 - 36 = 36$$

$$\Rightarrow \quad BD = 6 \text{ m}$$

2. In right ΔADB, $\sin A = \frac{BD}{AB} = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}}$ (from part (1))

$$\Rightarrow \quad \sin A = \sin 45° \Rightarrow \angle A = 45°$$

In right ΔBDC, $\tan C = \frac{BD}{DC} = \frac{6}{6}$

$\Rightarrow \tan C = 1 = \tan 45^\circ \Rightarrow \angle C = 45^\circ$

3. Here, $\sin A = \frac{1}{\sqrt{2}}$ and $\cos C = \cos 45^\circ = \frac{1}{\sqrt{2}}$

$\therefore \sin A + \cos C = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

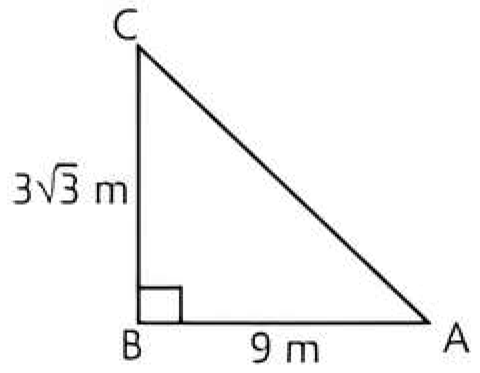
Or

Here, $\tan C = 1$ and $\tan A = \tan 45^\circ = 1$

$\Rightarrow \tan^2 C + \tan^2 A = 1 + 1 = 2$

Case Study 5

Three friends—Sanjeev, Amit and Digvijay are playing hide and seek in a park. Sanjeev and Amit were supposed to hide and Digvijay had to find both of them. If the positions of three friends are at A, B and C respectively as shown in the figure and forms a right-angled triangle such that AB = 9 m, BC = $3\sqrt{3}$ m and $\angle B = 90^\circ$.



Based on the above information, solve the following questions:

Q 1. Find the measure of $\angle A$ by using trigonometric ratio.

Q 2. Find the measure of $\angle C$ by using trigonometric ratio.

Q 3. Find the length of AC.

Q 4. Find the value of cos 2A.

Or

Find the value of $\sin\left(\frac{C}{2}\right)$.

Solutions

1. We have, AB = 9 m, BC = $3\sqrt{3}$ m

In right ΔABC, we have

$\tan A = \frac{BC}{AB} = \frac{3\sqrt{3}}{9} = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan A = \tan 30^\circ \Rightarrow \angle A = 30^\circ$

2. In right ΔABC,

We have, $\tan C = \frac{AB}{BC} = \frac{9}{3\sqrt{3}} = \sqrt{3}$

$\Rightarrow \tan C = \tan 60^\circ \Rightarrow \angle C = 60^\circ$

3. In right ΔABC, $\sin A = \frac{BC}{AC}$

$\Rightarrow \sin 30^\circ = \frac{BC}{AC}$ (from part (1))

$\Rightarrow \frac{1}{2} = \frac{3\sqrt{3}}{AC} \Rightarrow AC = 6\sqrt{3}$ m

4. $\because \angle A = 30^\circ$ (from part (1))

$\therefore \cos 2A = \cos(2 \times 30^\circ) = \cos 60^\circ = \frac{1}{2}$

Or

$\angle C = 60^\circ$

$\sin\left(\frac{C}{2}\right) = \sin\left(\frac{60^\circ}{2}\right) = \sin 30^\circ = \frac{1}{2}$

Very Short Answer Type Questions

Q 1. If $\sqrt{3}\sin\theta = \cos\theta$, then find the value of $\frac{3\cos^2\theta + 2\cos\theta}{3\cos\theta + 2}$ [CBSE 2015]

Q 2. Evaluate: $\sin^2 60^\circ + 2\tan 45^\circ - \cos^2 30^\circ$. [CBSE SQP 2023-24]

Q 3. Evaluate: $2(\sin^2 45^\circ + \cot^2 30^\circ) - 6(\cos^2 45^\circ - \tan^2 30^\circ)$ [CBSE 2023]

Q 4. Evaluate : $\frac{3}{2}\tan^2 30^\circ - 2\cos^2 90^\circ - \frac{1}{2}\text{cosec}^2 30^\circ$ [CBSE 2023]

Q 5. Evaluate: $\tan^2 60^\circ - 2\,\text{cosec}^2 30^\circ - 2\tan^2 30^\circ$ [CBSE 2023]

Q 6. Evaluate:

$$\frac{5}{\cos^2 30^\circ} + \frac{1}{\sin^2 60^\circ} - \cot^2 45^\circ + 2\sin^2 90^\circ.$$

[CBSE 2023]

Q 7. Evaluate $2\sec^2\theta + 3\,\text{cosec}^2\theta - 2\sin\theta\cos\theta$ if $\theta = 45$. [CBSE 2023]

Q 8. If $\sin x + \cos y = 1$, $x = 30^\circ$ and y is an acute angle, find the value of y. [CBSE 2019]

Q 9. Find the value of x:

$$2\,\text{cosec}^2 30^\circ + x\sin^2 60^\circ - \frac{3}{4}\tan^2 30^\circ = 10$$

[CBSE SQP 2023-24]

Q 10. If $4\cot^2 45^\circ - \sec^2 60^\circ + \sin^2 60^\circ + p = \frac{3}{4}$, then find the value of p. [CBSE 2023]

Q 11. If $\sin\theta - \cos\theta = 0$, then find the value of $\sin^4\theta + \cos^4\theta$. [CBSE 2023, 17]

Q 12. If θ is an acute angle and $\sin\theta = \cos\theta$, find the value of $\tan^2\theta + \cot^2\theta - 2$. [CBSE 2023]

Q 13. Find the value of $(\text{cosec}^2\theta - 1)\cdot\tan^2\theta$. [CBSE 2016]

Q 14. If $\sec\theta + \tan\theta = 7$, then evaluate $\sec\theta - \tan\theta$.

Q 15. If $\tan\theta = \frac{a}{x}$, find the value of $\frac{x}{\sqrt{a^2 + x^2}}$.

Q 16. If $\cos A + \cos^2 A = 1$, then find the value of $\sin^2 A + \sin^4 A$. [CBSE 2023]

Short Answer Type-I Questions

Q 1. If $\cot\theta = \frac{15}{8}$, then evaluate $\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$.

[CBSE 2016]

Q 2. If $\tan(A+B) = \sqrt{3}$ and $(A-B) = \frac{1}{\sqrt{3}}$;

$0° < A+B \le 90°, A > B$, then find A and B.

[NCERT EXERCISE; CBSE SQP 2023-24; CBSE 2016]

Q 3. If $\sin(A+B) = 1$ and $\cos(A-B) = \sqrt{3}/2$, $< A+B \le 90°, 0°$ and $A > B$, then find the measures of angles A and B. [CBSE SQP 2022-23]

Q 4. Prove that $1 + \frac{\cot}{1+\text{cosec}\,\alpha} = \text{cosec}\,\alpha$.

[NCERT EXEMPLAR; CBSE 2020]

Q 5. Prove that $2\cos^2\theta + \frac{2}{1+\cot^2\theta} = 2.$ [V. Imp.]

Q 6. Find an acute angle θ when $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$

Q 7. Prove that: $\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A.$ [CBSE 2023]

Q 8. Prove that $(\sec^4\theta - \sec^2\theta) = (\tan^2\theta + \tan^4\theta)$.

[NCERT EXEMPLAR; V. Imp., CBSE 2020]

Q 9. Prove that $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A.$

[NCERT EXERCISE; CBSE 2015]

Q 10. Prove that [CBSE 2023; CBSE SQP 2023-24]

$\sec\theta\,(1-\sin\theta)(\sec\theta + \tan\theta) = 1.$

Short Answer Type-II Questions

Q 1. If $4\tan\theta = 3$, evaluate $\left(\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}\right)$.

[CBSE 2018]

Q 2. If $\sin\theta = \frac{12}{13}, 0° < \theta < 90°$, find the value of

$\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta\cdot\cos\theta} \times \frac{1}{\tan^2\theta}$.

[CBSE 2015]

Q 3. Prove that: $\frac{1+\sec\theta}{\sec\theta} = \frac{\sin^2\theta}{1-\cos\theta}$. [CBSE 2023]

Q 4. Prove that: $\frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta} = 2$.

[V. Imp.]

Q 5. Prove that: [CBSE 2023]

$$\left(1+\frac{1}{\tan^2 A}\right)\left(1+\frac{1}{\cot^2 A}\right) = \frac{1}{\sin^2 A - \sin^4 A}$$

Q 6. Prove that $(\sin A + \text{cosec}\,A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$. [NCERT EXERCISE; CBSE 2016, 19, 23]

Q 7. Prove that: [CBSE 2023]

$$\frac{1-\cos\theta}{1+\cos\theta} = (\text{cosec}\,\theta - \cot\theta)^2$$

OR

Prove that: [CBSE SQP 2023-24]

$$(\text{cosec}\,\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$$

Q 8. Prove that: $(\text{cosec}\,\theta + \cot\theta)^2 = \frac{\sec\theta + 1}{\sec\theta - 1}$.

[CBSE 2016]

Q 9. Prove that:

$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\,\text{cosec}\,\theta$ [CBSE 2023]

Q 10. Prove that: $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A.$

[CBSE 2023]

Long Answer Type Questions

Q 1. If $m = \cos\theta - \sin\theta$ and $n = \cos\theta + \sin\theta$, then show that $\sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} = \frac{2}{\sqrt{1-\tan^2\theta}}$. [CBSE 2016]

Q 2. If $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$, then find the value of $\cot^2\theta + \tan^2\theta$.

Q 3. Prove that: $\frac{\cos^2}{1-\tan\theta} + \frac{\sin^3\theta}{\sin\theta - \cos\theta} = 1 + \sin\theta\cdot\cos\theta.$

[CBSE 2017]

Q 4. Prove that: $\frac{\sin\theta}{\cot\theta + \text{cosec}\,\theta} = 2 + \frac{\sin\theta}{\cot\theta - \text{cosec}\,\theta}$.

Q 5. Prove that:

$\sin^2\theta\tan\theta + \cos^2\theta\cot\theta + 2\sin\theta\cos\theta = \tan\theta + \cot\theta.$

[CBSE 2016]

Q 6. If $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ and $\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1$,

prove that: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2.$ [CBSE 2017]

Q 7. Prove the following trigonometric identity:

$\sin A(1+\tan A) + \cos A(1+\cos A) = \sec A + \text{cosec}\,A.$

[CBSE 2015]

Solutions

Very Short Answer Type Questions

1. $\dfrac{3\cos^2\theta+2\cos\theta}{3\cos\theta+2}$

$= \dfrac{3\times(\sqrt{3}\sin\theta)^2+2\times\sqrt{3}\sin\theta}{3\times\sqrt{3}\sin\theta+2}$ $(\because \cos\theta = \sqrt{3}\sin\theta)$

$= \dfrac{3\times 3\sin^2\theta+2\sqrt{3}\sin\theta}{3\sqrt{3}\sin\theta+2} = \dfrac{\sqrt{3}\sin\theta(3\sqrt{3}\sin\theta+2)}{(3\sqrt{3}\sin\theta+2)}$

$= \sqrt{3}\sin\theta$

2. $\sin^2 60^\circ + 2\tan 45^\circ - \cos^2 30^\circ$

$= \left(\dfrac{\sqrt{3}}{2}\right)^2 + 2(1) - \left(\dfrac{\sqrt{3}}{2}\right)^2$

$= \dfrac{3}{4}+2-\dfrac{3}{4} = 2.$

3. $2(\sin^2 45^\circ + \cot^2 30^\circ) - 6(\cos^2 45^\circ - \tan^2 30^\circ)$

$= 2\left\{\left(\dfrac{1}{\sqrt{2}}\right)^2 + (\sqrt{3})^2\right\} - 6\left\{\left(\dfrac{1}{\sqrt{2}}\right)^2 - \left(\dfrac{1}{\sqrt{3}}\right)^2\right\}$

$= 2\left(\dfrac{1}{2}+3\right) - 6\left(\dfrac{1}{2}-\dfrac{1}{3}\right)$

$= 2\times\dfrac{7}{2} - 6\times\dfrac{1}{6} = 7-1 = 6.$

4. $\dfrac{3}{2}\tan^2 30^\circ - 2\cos^2 90^\circ - \dfrac{1}{2}\operatorname{cosec}^2 30^\circ$

$= \dfrac{3}{2}\times\left(\dfrac{1}{\sqrt{3}}\right)^2 - 2\times(0)^2 - \dfrac{1}{2}\times(2)^2$

$= \dfrac{3}{2}\times\dfrac{1}{3} - 2\times 0 - \dfrac{1}{2}\times 4$

$= \dfrac{1}{2} - 0 - 2 = \dfrac{-3}{2}.$

5. $\tan^2 60^\circ - 2\operatorname{cosec}^2 30^\circ - 2\tan^2 30^\circ$

$= (\sqrt{3})^2 - 2\times(2)^2 - 2\times\left(\dfrac{1}{\sqrt{3}}\right)^2$

$= 3 - 2\times 4 - 2\times\dfrac{1}{3}$

$= 3 - 8 - \dfrac{2}{3} = -5 - \dfrac{2}{3} = -\dfrac{17}{3}.$

6. $\dfrac{5}{\cot^2 30^\circ} + \dfrac{1}{\sin^2 60^\circ} - \cot^2 45^\circ + 2\sin^2 90^\circ$

$= \dfrac{5}{(\sqrt{3})^2} + \dfrac{1}{(\sqrt{3}/2)^2} - (1)^2 + 2\times(1)^2$

$= \dfrac{5}{3} + \dfrac{1}{(3/4)} - 1 + 2 = \dfrac{5}{3} + \dfrac{4}{3} + 1$

$= \dfrac{9}{3} + 1 = 3 + 1 = 4.$

7. $2\sec^2\theta + 3\operatorname{cosec}^2\theta - 2\sin\theta\cdot\cos\theta$

$= 2\sec^2 45^\circ + 3\operatorname{cosec}^2 45^\circ - 2\sin 45^\circ\cdot\cos 45^\circ$ (at $\theta = 45^\circ$)

$= 2(\sqrt{2})^2 + 3(\sqrt{2})^2 - 2\times\dfrac{1}{\sqrt{2}}\times\dfrac{1}{\sqrt{2}}$

$= 2\times 2 + 3\times 2 - 2\times\dfrac{1}{2} = 4 + 6 - 1 = 10 - 1 = 9.$

8. Given, $\sin x + \cos y = 1$

Put $x = 30^\circ$.

$\sin 30^\circ + \cos y = 1 \Rightarrow \cos y = 1 - \dfrac{1}{2}$ $(\because \sin 30^\circ = 1/2)$

$\Rightarrow \cos y = \dfrac{1}{2} \Rightarrow y = 60^\circ$ $(\because \cos 60^\circ = 1/2)$

9. $2\operatorname{cosec}^2 30^\circ + x\sin^2 60^\circ - \dfrac{3}{4}\tan^2 30^\circ = 10$

$\Rightarrow 2\times(2)^2 + x\left(\dfrac{\sqrt{3}}{2}\right)^2 - \dfrac{3}{4}\times\left(\dfrac{1}{\sqrt{3}}\right)^2 = 10$

$\Rightarrow 2\times 4 + x\times\dfrac{3}{4} - \dfrac{3}{4}\times\dfrac{1}{3} = 10$

$\Rightarrow 8 + \dfrac{3x}{4} - \dfrac{1}{4} = 10$

$\Rightarrow \dfrac{3x}{4} = 10 - 8 + \dfrac{1}{4}$

$= 2 + \dfrac{1}{4} = \dfrac{9}{4}$

$\Rightarrow x = \dfrac{9}{3} = 3$

10. $4\cot^2 45^\circ - \sec^2 60^\circ + \sin^2 60^\circ + p = \dfrac{3}{4}$

$\Rightarrow 4\times(1)^2 - (2)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2 + p = \dfrac{3}{4}$

$\Rightarrow 4\times 1 - 4 + \dfrac{3}{4} + p = \dfrac{3}{4}$

$\Rightarrow p = \dfrac{3}{4} - \dfrac{3}{4} + 4 - 4 = 0$

11. Given, $\sin\theta - \cos\theta = 0 \Rightarrow \sin\theta = \cos\theta$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} = 1$

$\Rightarrow \tan\theta = \tan 45^\circ \Rightarrow \theta = 45^\circ$

$\therefore \sin^4\theta + \cos^4\theta = \sin^4 45^\circ + \cos^4 45^\circ$

$= \left(\dfrac{1}{\sqrt{2}}\right)^4 + \left(\dfrac{1}{\sqrt{2}}\right)^4$

$= \dfrac{1}{4} + \dfrac{1}{4} = \dfrac{1}{2}$

$(\because \tan 45^\circ = 1$ and $\sin 45^\circ = \cos 45^\circ = 1/\sqrt{2})$

12. Given, $\sin\theta = \cos\theta$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = 1$

$\Rightarrow \tan\theta = \tan 45^\circ \Rightarrow \theta = 45^\circ$.

$\therefore \tan^2\theta + \cot^2\theta - 2 = \tan^2 45^\circ + \cot^2 45^\circ - 2$

$= (1)^2 + (1)^2 - 2$

$= 1 + 1 - 2 = 2 - 2 = 0$.

13. $(\text{cosec}^2\theta - 1)\cdot\tan^2\theta = \cot^2\theta\cdot\tan^2\theta$

$= \cot^2\theta\cdot\frac{1}{\cot^2\theta} = 1$

$\left(\because \text{cosec}^2\theta - 1 = \cot^2\theta \text{ and } \tan\theta = \frac{1}{\cot\theta}\right)$

14. Given, $\sec\theta + \tan\theta = \frac{7}{1}$

$\Rightarrow \frac{(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)}{(\sec\theta-\tan\theta)} = \frac{7}{1}$

$\Rightarrow \sec\theta - \tan\theta = \frac{\sec^2\theta - \tan^2\theta}{7}$

$[\because (a+b)(a-b) = a^2 - b^2]$

$\therefore \sec\theta - \tan\theta = \frac{1}{7}$ $(\because \sec^2\theta - \tan^2\theta = 1)$

15. $\frac{x}{\sqrt{a^2+x^2}} = \frac{x}{x\sqrt{\left(\frac{a}{x}\right)^2+1}} = \frac{1}{\sqrt{\left(\frac{a}{x}\right)^2+1}}$

$= \frac{1}{\sqrt{\tan^2\theta+1}}$ $\left(\because \tan\theta = \frac{a}{x}\right)$

$= \frac{1}{\sqrt{\sec^2\theta}} = \frac{1}{\sec\theta} = \cos\theta$ $(\because \tan^2\theta + 1 = \sec^2\theta)$

16. Given, $\cos A + \cos^2 A = 1$

$\Rightarrow \cos A = 1 - \cos^2 A$

$\Rightarrow \cos A = \sin^2 A$ $(\because \sin^2 A + \cos^2 A = 1)$

Squaring on both sides, we get

$\cos^2 A = \sin^4 A$

$\Rightarrow 1 - \sin^2 A = \sin^4 A$ $(\because \cos^2 A = 1 - \sin^2 A)$

$\Rightarrow \sin^2 A + \sin^4 A = 1$.

Short Answer Type-I Questions

1. Given expression $= \frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$

$= \frac{2(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)2(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta}$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= \cot^2\theta = \left(\frac{15}{8}\right)^2 = \frac{225}{64}$ $\left(\because \cot\theta = \frac{15}{8}\right)$

2. Given, $\tan(A+B) = \sqrt{3}$

$\Rightarrow \tan(A+B) = \tan 60^\circ$ $(\because \tan 60^\circ = \sqrt{3})$

$\Rightarrow A + B = 60^\circ$ ...(1)

Again, $\tan(A-B) = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan(A-B) = \tan 30^\circ$ $(\because \tan 30^\circ = 1/\sqrt{3})$

$\Rightarrow A - B = 30^\circ$ ...(2)

Adding eqs. (1) and (2), we get

$2A = 90^\circ \Rightarrow A = 45^\circ$

From eq. (1), we get

$45^\circ + B = 60^\circ \Rightarrow B = 15^\circ$

Hence, $\angle A = 45^\circ$ and $\angle B = 15^\circ$

COMMON ERR(!)R

Some students confused between values of tan 30° and tan 60°. They take wrong values as tan 30° = $\sqrt{3}$ and tan 60° = $\frac{1}{\sqrt{3}}$.

3. Given,

$\sin(A+B) = 1 = \sin 90^\circ$

So, $A + B = 90^\circ$ ...(1)

and $\cos(A-B) = \frac{\sqrt{3}}{2} = \cos 30^\circ$ ...(2)

$A - B = 30^\circ$

Adding eqs. (1) and (2),

$2\angle A = 120^\circ \Rightarrow \angle A = 60^\circ$

Put $\angle A = 60^\circ$ in eq. (1),

$\angle B = 30^\circ$

4. LHS $= 1 + \frac{\cot^2\alpha}{1+\text{cosec}\,\alpha}$

$= 1 + \frac{\text{cosec}^2\alpha - 1}{1+\text{cosec}\,\alpha}$ $(\because \text{cosec}^2\theta + \cot^2\theta = 1)$

$= 1 + \frac{(\text{cosec}\,\alpha+1)(\text{cosec}\,\alpha-1)}{(1+\text{cosec}\,\alpha)}$

$[\because a^2 - b^2 = (a+b)(a-b)]$

$= 1 + \text{cosec}\,\alpha - 1 = \text{cosec}\,\alpha$

= RHS **Hence proved.**

COMMON ERR(!)R

Sometimes students don't apply this formula: $(a^2 - b^2) = (a+b)(a-b)$. They directly simplify equation which leads to incorrect result.

5. LHS $= 2\cos^2\theta + \frac{2}{1+\cot^2\theta}$

$= 2\cos^2\theta + \frac{2}{\text{cosec}^2\theta}$ $(\because 1 + \cot^2\theta = \text{cosec}^2\theta)$

$= 2\cos^2\theta + 2\sin^2\theta$

$= 2(\cos^2\theta + \sin^2\theta)$ $(\because \cos^2\theta + \sin^2\theta = 1)$

$= 2 \times 1 = 2 =$ RHS **Hence proved.**

6. Given, $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$

Dividing the numerator and denominator of LHS by $\cos\theta$.

$\frac{1-\tan\theta}{1+\tan\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$

Comparing both sides, $\tan\theta = \sqrt{3}$

or $\theta = 60^\circ$

7. LHS $= \dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$

$= \dfrac{\sin A\,\{1 - 2\sin^2 A\}}{\cos A\,\{2\cos^2 A - 1\}}$ $(\because \sin^2 A = \cos^2 A = 1)$

$= \dfrac{\sin A\,\{1 - 2\sin^2 A\}}{\cos A\,\{2(1 - \sin^2 A) - 1\}}$

$= \dfrac{\sin A\,(1 - 2\sin^2 A)}{\cos A\,\{2 - 2\sin^2 A - 1\}} = \dfrac{\sin A\,(1 - 2\sin^2 A)}{\cos A\,(1 - 2\sin^2 A)}$

$= \dfrac{\sin A}{\cos A} = \tan A = \text{RHS}$ **Hence Proved.**

8. LHS $= \sec^4\theta - \sec^2\theta$

$= \sec^2\theta\,(\sec^2\theta - 1)$

TR!CK

$\because$ *$\sec^2\theta - \tan^2\theta = 1$*

$\therefore$ *$1 + \tan^2\theta = \sec^2\theta$*

or *$\tan^2\theta = \sec^2\theta - 1$*

$= (1 + \tan^2\theta)\tan^2\theta$

$= \tan^2\theta + \tan^4\theta = \text{RHS}$ **Hence proved.**

9. LHS $= \sqrt{\dfrac{1 + \sin A}{1 - \sin A}}$

TiP

Follow step-by-step simplification to avoid errors.

$= \sqrt{\dfrac{(1 + \sin A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)}}$

$= \dfrac{(1 + \sin A)}{\sqrt{1 - \sin^2 A}} = \dfrac{1 + \sin A}{\sqrt{\cos^2 A}}$ $(\because \sin^2 A + \cos^2 A = 1)$

$= \dfrac{1 + \sin A}{\cos A} = \dfrac{1}{\cos A} + \dfrac{\sin A}{\cos A} = \sec A + \tan A$

$= \text{RHS}$ **Hence proved.**

10. LHS $= \sec\theta\,(1 - \sin\theta)\,(\sec A + \tan\theta)$

$= \sec\theta \cdot (1 - \sin\theta)\left(\dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}\right)$

$= \sec\theta \cdot (1 - \sin\theta) \cdot \dfrac{(1 + \sin\theta)}{\cos\theta}$

$= \sec\theta \cdot \dfrac{(1 - \sin^2\theta)}{\cos\theta}$ $[\because (a - b)(a + b) = a^2 - b^2]$

$= \sec\theta \cdot \dfrac{\cos^2\theta}{\cos\theta}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

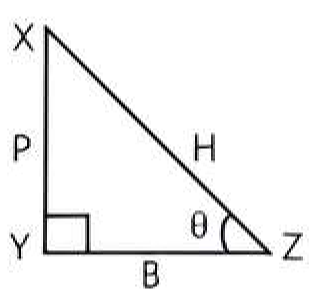
$= \dfrac{1}{\cos\theta} \times \cos\theta = 1 = \text{RHS}$ **Hence Proved**

Short Answer Type-II Questions

1. Let triangle XYZ is a right-angled triangle in which $\angle Y = 90°$.

Given, $4\tan\theta = 3$

$\Rightarrow \tan\theta = \dfrac{3}{4} = \dfrac{P}{B} = \dfrac{XY}{YZ}$



Let $P = 3k$ and $B = 4k$, where k is a positive number.

In right-angled ΔXYZ,

$(XZ)^2 = (XY)^2 + (YZ)^2$

(by Pythagoras theorem)

$\Rightarrow H^2 = P^2 + B^2 = (3k)^2 + (4k)^2$

$= 9k^2 + 16k^2 = 25k^2$

$\Rightarrow$ $H = 5k$

TR!CK

In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

So, $\sin\theta = \dfrac{P}{H} = \dfrac{3k}{5k} = \dfrac{3}{5}$ and $\cos\theta = \dfrac{B}{H} = \dfrac{4k}{5k} = \dfrac{4}{5}$

$\therefore \dfrac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1} = \dfrac{4\left(\frac{3}{5}\right) - \frac{4}{5} + 1}{4\left(\frac{3}{5}\right) + \frac{4}{5} - 1} = \dfrac{\frac{12 - 4 + 5}{5}}{\frac{12 + 4 - 5}{5}}$

$= \dfrac{12 - 4 + 5}{12 + 4 - 5} = \dfrac{17 - 4}{16 - 5} = \dfrac{13}{11}$

2. Given, $\sin\theta = \dfrac{12}{13}$

TiP

Here, we have given only the value of $\sin\theta$. So, we have to convert the given expression into $\sin\theta$ with the help of suitable identities.

$\therefore \dfrac{\sin^2\theta - \cos^2\theta}{2\sin\theta\cos\theta} \times \dfrac{1}{\tan^2\theta}$

$= \dfrac{\sin^2\theta - (1 - \sin^2\theta)}{2\sin\theta\cos\theta} \times \dfrac{\cos^2}{\sin^2\theta}$

$= \dfrac{2\sin^2\theta - 1}{2\sin^3\theta}.\cos\theta = \dfrac{2\sin^2\theta - 1}{2\sin^3\theta}.\sqrt{1 - \sin^2\theta}$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= \dfrac{2\left(\frac{12}{13}\right)^2 - 1}{2\left(\frac{12}{13}\right)^3}.\sqrt{1 - \left(\frac{12}{13}\right)^2} = \dfrac{2 \times \frac{144}{169} - 1}{2 \times \frac{12}{13} \times \frac{144}{169}} \times \sqrt{1 - \frac{144}{169}}$

$= \dfrac{\frac{288 - 169}{169}}{\frac{24}{13} \times \frac{144}{169}} \times \sqrt{\dfrac{25}{169}} = \dfrac{119}{169} \times \dfrac{13 \times 169}{24 \times 144} \times \dfrac{5}{13}$

$= \dfrac{119 \times 5}{24 \times 144} = \dfrac{595}{3456}$.

3. LHS $= \dfrac{1 + \sec\theta}{\sec\theta} = \dfrac{1 + 1/\cos\theta}{1/\cos\theta}$

$= \dfrac{\cos\theta + 1}{\cos\theta} \times \cos\theta = (1 + \cos\theta) \times \dfrac{(1 - \cos\theta)}{(1 - \cos\theta)}$

$= \dfrac{1 - \cos^2\theta}{1 - \cos\theta} = \dfrac{\sin^2\theta}{1 - \cos\theta}$ $(\because \sin^2\theta + \cos^2\theta = 1)$

$= \text{RHS}$

4. LHS $= \dfrac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \dfrac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$

TiP

Adequate practice of identities is necessary to avoid errors in simplification.

$$= \frac{(\cos\theta+\sin\theta)(\cos^2\theta+\sin^2\theta-\cos\theta\sin\theta)}{(\cos\theta+\sin\theta)}$$

$$+ \frac{(\cos\theta-\sin\theta)(\cos^2\theta+\sin^2\theta+\cos\theta\sin\theta)}{(\cos\theta-\sin\theta)}$$

$$= (1-\cos\theta\sin\theta)+(1+\cos\theta\sin\theta)$$

$(\because \sin^2\theta+\cos^2\theta = 1)$

$= 1+1 = 2 =$ RHS **Hence proved.**

COMMON ERROR

Sometimes students don't apply these formulae:

$a^3+b^3 = (a+b)(a^2-ab+b^2)$

and $a^3-b^3 = (a-b)(a^2+ab+b^2)$

They directly simplify equation which leads to incorrect result.

5. LHS $= \left(1+\frac{1}{\tan^2 A}\right)\left(1+\frac{1}{\cot^2 A}\right)$

TiP

Adequate practice of identities is necessary to avoid errors in simplification.

$= (1+\cot^2 A)(1+\tan^2 A)$ $(\because \tan A\cdot\cot A = 1)$

$= \operatorname{cosec}^2 A\cdot\sec^2 A = \frac{1}{\sin^2 A}\cdot\frac{1}{\cos^2 A}$

$(\because \operatorname{cosec}^2\theta-\cot^2\theta = 1$ and $\sec^2\theta-\tan^2\theta = 1)$

$= \frac{1}{\sin^2 A(1-\sin^2 A)}$ $(\because \sin^2 A+\cos^2 A = 1)$

$= \frac{1}{\sin^2 A-\sin^4 A} =$ RHS **Hence proved.**

6. LHS $= (\sin A+\operatorname{cosec} A)^2+(\cos A+\sec A)^2$

$= \sin^2 A+\operatorname{cosec}^2 A+2\sin A\operatorname{cosec} A+\cos^2 A + \sec^2 A+2\cos A\sec A$

$[\because (a+b)^2 = a^2+b^2+2ab]$

$= (\sin^2 A+\cos^2 A)+(\operatorname{cosec}^2 A+\sec^2 A) + 2\sin A\left(\frac{1}{\sin A}\right)+\cos A\left(\frac{1}{\cos A}\right)$

$= 1+(1+\cot^2 A+1+\tan^2 A)+2+2$

$(\because \operatorname{cosec}^2 A = 1+\cot^2 A$ and $\sec^2 A = 1+\tan^2 A)$

$= 7+\tan^2 A+\cot^2 A =$ RHS **Hence proved.**

7. LHS $= \frac{1-\cos\theta}{1+\cos\theta}$

$$= \frac{1-\cos\theta}{1+\cos\theta}\times\frac{1-\cos\theta}{1-\cos\theta}$$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} = \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$[\because \sin^2\theta+\cos^2\theta = 1]$

$$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2 = \left(\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\right)^2$$

$= [\operatorname{cosec}\theta-\cot\theta]^2 =$ RHS

OR

RHS $= (\operatorname{cosec}\theta-\cot\theta)^2$

$$= \left(\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta} = \frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}$$ $[\because \sin^2\theta+\cos^2\theta = 1]$

$$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$ $[\because a^2-b^2 = (a-b)(a+b)]$

$$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)} = \text{LHS}$$

$$= \frac{(1-\cos\theta)}{(1+\cos\theta)}$$

8. LHS $= (\operatorname{cosec}\theta+\cot\theta)^2$

$$= \left(\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{1+\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1+\cos\theta)^2}{\sin^2} = \frac{(1+\cos\theta)^2}{1-\cos^2\theta}$$ $(\because \sin^2\theta+\cos^2\theta = 1)$

$$= \frac{(1+\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)}$$ $(\because a^2-b^2 = (a+b)(a-b))$

$$= \frac{1+\cos\theta}{1-\cos\theta} = \frac{1+\frac{1}{\sec\theta}}{1-\frac{1}{\sec\theta}}$$ $\left(\because \cos\theta = \frac{1}{\sec\theta}\right)$

$$= \frac{\frac{\sec\theta+1}{\sec\theta}}{\frac{\sec\theta-1}{\sec\theta}} = \frac{\sec\theta+1}{\sec\theta-1} = \text{RHS}$$ **Hence proved.**

9. LHS $= \frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}$

$$= \frac{\sin^2\theta+(1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$$

$$= \frac{\sin^2\theta+1+\cos\theta^2+2\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{(\sin^2\theta+\cos^2\theta)+1+2\cos\theta}{\sin\theta(1+\cos\theta)}$$

$(\because \sin^2\theta+\cos^2\theta = 1)$

$$= \frac{1+1+2\cos\theta}{\sin\theta(1+\cos\theta)} = \frac{2+2\cos\theta}{(1+\cos\theta)\cdot\sin\theta}$$

$$= \frac{2(1+\cos\theta)}{(1+\cos\theta)\sin\theta} = 2\cdot\frac{1}{\sin\theta} = 2\operatorname{cosec}\theta = \text{RHS}$$

Hence proved

10. LHS $= \frac{\cos A}{1+(\sin A)}+\frac{(1+\sin A)}{\cos A}$

$$= \frac{\cos A}{(1+\sin A)}\times\frac{1-\sin A}{(1-\sin A)}+\frac{(1+\sin A)}{\cos A}$$

$$= \frac{\cos A(1-\sin A)}{(1-\sin^2 A)}+\frac{(1+\sin A)}{\cos A}$$

$$= \frac{\cos A(1-\sin A)}{\cos^2 A}+\frac{(1+\sin A)}{\cos A}$$ $[\therefore \sin^2 A+\cos^2 A = 1]$

$$= \frac{(1-\sin A)}{\cos A} + \frac{(1+\sin A)}{\cos A}$$

$$= \frac{1-\sin A + 1+\sin A}{\cos A} = 2\times\frac{1}{\cos A}$$

$$= 2 \sec A = \text{RHS}$$

Long Answer Type Questions

1. $\text{LHS} = \sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}}$

$$= \sqrt{\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}} + \sqrt{\frac{\cos\theta+\sin\theta}{\cos\theta+\sin\theta}}$$

$$= \frac{\sqrt{(\cos\theta-\sin\theta)^2} + \sqrt{(\cos\theta+\sin\theta)^2}}{\sqrt{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}}$$

$$= \frac{\cos\theta-\sin\theta+\cos\theta+\sin\theta}{\sqrt{\cos^2\theta-\sin^2\theta}}$$

$[\because (a+b)(a-b) = a^2 - b^2]$

$$= \frac{2\cos\theta}{\sqrt{\cos^2\theta-\sin^2\theta}}$$

$$= \frac{2\frac{\cos\theta}{\cos\theta}}{\sqrt{\frac{\cos^2\theta}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}}}$$

(divide numerator and denominator by $\cos\theta$)

$$= \frac{2}{\sqrt{1-\tan^2\theta}} = \text{RHS}$$

Hence proved.

2. Given, $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$

TR!CK

$\because \quad \sqrt{3}\times\sqrt{3} = 3$

$\therefore \quad 3 = 3\times 1$

Here we take 3 and 1 as a factors of 3.

So, middle term $-4 = -3 - 1$.

$$\sqrt{3}\cot^2\theta - 3\cot\theta - \cot\theta + \sqrt{3} = 0$$

$$\Rightarrow \sqrt{3}\cot\theta(\cot\theta-\sqrt{3}) - 1(\cot\theta-\sqrt{3}) = 0$$

$$\Rightarrow (\cot\theta-\sqrt{3})(\sqrt{3}\cot\theta-1) = 0$$

$$\Rightarrow \cot\theta-\sqrt{3} = 0 \quad\text{or}\quad \cot\theta-\sqrt{3} = 0$$

$$\Rightarrow \cot\theta = \sqrt{3} \quad\text{or}\quad \cot\theta = \frac{1}{\sqrt{3}}$$

When $\cot^2\theta = \sqrt{3}$, $\tan\theta = \frac{1}{\sqrt{3}}$

$$\therefore \cot^2 + \tan^2\theta = (\sqrt{3})^2 + \left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 3 + \frac{1}{3} = \frac{9+1}{3} = \frac{10}{3}$$

When $\cot\theta = \frac{1}{\sqrt{3}}$, $\tan\theta = \sqrt{3}$

$$\therefore \cot^2\theta + \tan^2\theta = \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2$$

$$= \frac{1}{3} + 3 = \frac{1+9}{3} = \frac{10}{3}$$

3. $\text{LHS} = \frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^3\theta}{\sin\theta-\cos\theta}$

TiP

Learn basic identities like $(a-b)^2$, $(a+b)^2$, (a^2-b^2), (a^3-b^3), (a^3+b^3), *etc.*

$$= \frac{\cos^2\theta}{1-\left(\frac{\sin\theta}{\cos\theta}\right)} + \frac{\sin^3\theta}{\sin\theta-\cos\theta}$$

$$= \frac{\cos^3\theta}{\cos\theta-\sin\theta} + \frac{\sin^3\theta}{\sin\theta-\cos\theta} = \frac{\cos^3\theta-\sin^3\theta}{(\cos\theta-\sin\theta)}$$

$$= \frac{(\cos\theta-\sin\theta)(\cos^2\theta+\sin^2\theta+\sin\theta\cdot\cos\theta)}{(\cos\theta-\sin\theta)}$$

$[\because a^3 - b^3 = (a-b)(a^2+ab+b^2)]$

$$= (\sin^2\theta + \cos^2\theta) + \sin\theta\cdot\cos\theta$$

$$= 1 + \sin\theta\cdot\cos\theta = \text{RHS} \qquad (\because \sin^2\theta + \cos^2\theta = 1)$$

Hence proved.

COMMON ERR(!)R

Sometimes students don't apply this formula:

$$a^3 - b^3 = (a-b)(a^2+ab+b^2).$$

They directly simplify equation which leads to incorrect result.

4. $\text{LHS} = \frac{\sin\theta}{\cot\theta+\operatorname{cosec}\theta} = \frac{\sin\theta}{\left(\frac{\cos\theta}{\sin\theta}+\frac{1}{\sin\theta}\right)} = \frac{\sin^2\theta}{\cos\theta+1}$

$$= \frac{1-\cos^2\theta}{1+\cos\theta}\ \frac{(1-\cos\theta)(1+\cos\theta)}{(1+\cos\theta)} = 1-\cos\theta$$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$$\text{RHS} = 2 + \frac{\sin\theta}{\cot\theta-\operatorname{cosec}\theta} = 2 + \frac{\sin\theta}{\left(\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)}$$

$$= 2 + \frac{\sin^2\theta}{\cos\theta-1} = 2 + \frac{1-\cos^2\theta}{\cos\theta-1}$$

$$= \frac{(1-\cos\theta)(1+\cos\theta)}{(1-\cos\theta)} = 2-(1+\cos\theta)$$

$$= 2 - 1 - \cos\theta = 1-\cos\theta$$

So, LHS = RHS **Hence proved.**

5. $\text{LHS} = \sin^2\theta\tan\theta + \cos^2\theta\cot\theta + 2\sin\theta\cos\theta$

TiP

Do simplify only one side at a time.

$$= \sin^2\theta\frac{\sin\theta}{\cos\theta} + \cos^2\theta\cdot\frac{\cos\theta}{\sin\theta} + 2\sin\theta\cos\theta$$

$$= \frac{\sin^4\theta+\cos^4\theta+2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{(\sin^2\theta+\cos^2\theta)^2}{\cos\theta\sin\theta} \qquad [\because a^2+b^2+2ab = (a+b)^2]$$

$$= \frac{1^2}{\cos\theta \sin\theta} = \frac{1}{\cos\theta \sin\theta} \qquad (\because \sin^2\theta + \cos^2\theta = 1)$$

RHS = $\tan\theta + \cot\theta$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{(\sin^2\theta + \cos^2\theta)}{\cos\theta \sin\theta} = \frac{1}{\cos\theta \sin\theta}$$

$(\because \sin^2\theta + \cos^2\theta = 1)$

So, LHS = RHS

Hence proved.

6. Given, $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ ...(1)

and $\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1$ ...(2)

TiP

Adequate practice of identities is necessary to avoid errors in simplification.

Squaring eqs. (1) and (2) and then adding, we get

$$\left(\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta\right)^2 + \left(\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta\right)^2 = 1+1$$

$$\Rightarrow \frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + \frac{2xy}{ab}\sin\theta\cos\theta$$

$$+ \frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta - \frac{2xy}{ab}\sin\theta\cos\theta = 2$$

$$\Rightarrow \frac{x^2}{a^2}(\sin^2\theta + \cos^2\theta) + \frac{y^2}{b^2}(\sin^2\theta + \cos^2\theta) = 2$$

$$\Rightarrow \frac{x^2}{a^2} \times 1 + \frac{y^2}{b^2} \times 1 = 2 \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$(\because \sin^2\theta + \cos^2\theta = 1)$

Hence proved.

COMMON ERROR

Sometimes students don't apply these formulae: $(a+b)^2 = a^2 + b^2 + 2ab$ and $(a-b)^2 = a^2 + b^2 - 2ab$. They directly simplify equation which leads to incorrect result.

7. LHS = $\sin A(1 + \tan A) + \cos A(1 + \cot A)$

TiP

Follow step-by-step simplification to avoid errors.

$$= \sin A\left(1 + \frac{\sin A}{\cos A}\right) + \cos A\left(1 + \frac{\cos A}{\sin A}\right)$$

$$= \sin A + \frac{\sin^2 A}{\cos A} + \cos A + \frac{\cos^2 A}{\sin A}$$

$$= (\sin A + \cos A) + \frac{\sin^2 A}{\cos A} + \frac{\cos^2 A}{\sin A}$$

$$= (\sin A + \cos A) + \left(\frac{\sin^3 A + \cos^3 A}{\sin A \cdot \cos A}\right)$$

$$= (\sin A + \cos A) + \frac{(\sin A + \cos A)(\sin^2 A + \cos^2 A - \sin A \cdot \cos A)}{\sin A \cdot \cos A}$$

$[\because (a^3 + b^3) = (a+b)(a^2 - ab + b^2)]$

$$= (\sin A + \cos A)\left\{1 + \frac{(1 - \sin A \cdot \cos A)}{\sin A \cdot \cos A}\right\}$$

$(\because \sin^2 A + \cos^2 A = 1)$

$$= \frac{\sin A + \cos A}{\sin A \cdot \cos A}(\sin A \cdot \cos A + 1 - \sin A \cdot \cos A)$$

$$= \frac{\sin A + \cos A}{\sin A \cdot \cos A} \times 1 = \frac{\sin A}{\sin A \cdot \cos A} + \frac{\cos A}{\sin A \cdot \cos A}$$

$$= \frac{1}{\cos A} + \frac{1}{\sin A} = \sec A + \operatorname{cosec} A = \text{RHS}$$

Hence proved.

COMMON ERROR

Sometimes students don't apply this formula: $(a^3 + b^3) = (a+b)(a^2 - ab + b^2)$. They directly simplify equation which leads to incorrect result.

Chapter Test

Multiple Choice Questions

Q 1. If cosec θ = $\sqrt{10}$, then sec θ is equal to:

a. $\frac{\sqrt{10}}{3}$ b. $\sqrt{10}$

c. $\frac{3}{\sqrt{10}}$ d. $\frac{2}{\sqrt{10}}$

Q 2. The value of (sin 30° + cos 30°) − (sin 60° + cos 60°) is:

a. −1 b. 0

c. 1 d. 2

Assertion and Reason Type Questions

Directions (Q.Nos. 3-4): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 3. Assertion (A): In ΔABC, right angled at B, if $\sin A = \frac{8}{17}$, then $\cos A = \frac{15}{17}$ and $\tan A = \frac{8}{15}$.

Reason (R): For acute angle θ, $\cos\theta = \frac{\text{Hypotenuse}}{\text{Base}}$, and $\tan\theta = \frac{\text{Base}}{\text{Perpendicular}}$.

Q 4. Assertion (A): If $\cos A + \cos^2 A = 1$, then $\sin^2 A + \sin^2 A = 2$.

Reason (R): $1 - \sin^2 A = \cos^2 A$, for any value of A.

Fill in the Blanks

Q 5. The maximum value of $\sin\theta$ is ....

Q 6. If $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$, then the value of $(\alpha + \beta)$ is ....

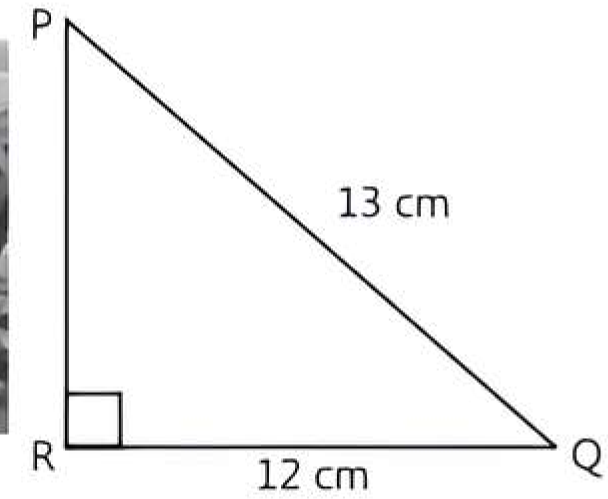
True/False

Q 7. $\sin^2\theta + \cos^2\theta = 1$ is a trigonometry identity.

Q 8. The value of $\cos\theta$ decreases from 1 to 0 when θ increases from $0°$ to $90°$.

Case Study Based Question

Q 9. Anika, a student of class 10th. She has to make a project on 'Introduction to Trigonometry'. She decides to make a bird house which is triangular in shape. She uses cardboard to make the bird house as shown in the figure. Consider the front side of bird house as a right angled triangle PQR, right angled at R.



Based on the given information, solve the following questions:

(i) If $\angle PQR = \theta$, then find the value of $\cos\theta$.

(ii) Find the value of $\sec\theta$.

(iii) Find the value of $\frac{\tan\theta}{1 + \tan^2\theta}$.

OR

Find the value of $\cot^2\theta - \csc^2\theta$.

Very Short Answer Type Questions

Q 10. Is it possible that $\sin\theta = \frac{15}{11}$?

Q 11. Simplify $9\sec^2\theta - 9\tan^2\theta$.

Short Answer Type-I Questions

Q 12. If $\sec\theta + \tan\theta = 9$, then find $(\sec\theta - \tan\theta)$.

Q 13. Prove that $(1 + \sin A)(\sec A - \tan A) = \cos A$.

Short Answer Type-II Questions

Q 14. Prove that : $\frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta} = 2$.

Q 15. Prove that

$$\cot^2 A\left(\frac{\sec A - 1}{\sin A + 1}\right) + \sec^2 A\left(\frac{\sin A - 1}{\sec A + 1}\right) = 0.$$

Long Answer Type Question

Q 16. Prove that $(1 + \cot A - \csc A)(1 + \tan A + \sec A) = 2$.